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7 SEISMIC LOADS Commentary

7.1 Estimation of Seismic Loads

Introduction

When earthquakes occur, a building undergoes dynamic motion. This is because the building is subjected to inertia forces that act in opposite direction to the acceleration of earthquake excitations. These inertia forces, called seismic loads, are usually dealt with by assuming forces external to the building. Since earthquake motions vary with time and inertia forces vary with time and direction, seismic loads are not constant in terms of time and space. In designing buildings, the maximum story shear force is considered to be the most influential, therefore in this chapter seismic loads are the static loads to give the maximum story shear force for each story, i.e. equivalent static seismic loads. Time histories of earthquake motions are also used to analyze high-rise buildings, and their elements and contents for seismic design. The earthquake motions for dynamic design are called design earthquake motions. In the previous recommendations, only the equivalent static seismic loads were considered to be seismic loads. In this chapter, not only equivalent static seismic loads but also design earthquake motions as time histories are included in seismic loads considered in the wider sense. In ISO/TC98 which deals with “bases for design of structures”, the term “action” is used instead of “load” and action includes not only load as external force but various influences that may cause deformations to the structures. In the future, “action” may take the place of “load”.

Section 7.1 deals with the fundamentals of estimating seismic loads. Sec.7.2 is about the calculation of seismic loads that gives equivalent static seismic loads for a building above the ground level. Sec.7.3 discusses the design earthquake motions in time histories that are used for dynamic analyses.

Overall framework and fundamental items and equations are described in each section as main text. The specific value of each factor is given in the main text when it is possible to give a normative value, or it is given in the commentary when only an informative value is possible.

Section 7.2 defines seismic loads as the story shear force for each story, and uses equivalent static seismic loads for limit state design, allowable stress design and ultimate strength design. As in the 1993 guidelines, the equivalent static seismic loads are story shear forces in the building that can be calculated through response spectrum analysis. The analytical model is fundamentally the multi-degree-of-freedom (MDOF) model (also called “lumped mass model”) that takes into account the soil-structure interaction considering sway and rocking motions. The maximum story shear force for each mode is calculated from the natural frequency and mode that can be calculated through eigenvalue analysis of the model, estimating the damping ratio for each mode. The seismic loads are the maximum story shear forces that can be calculated from the shear force of each mode. The fixed base model can be employed if the building is on a firm soil, so that soil-structure interaction may be neglected.

Since the response spectra are affected by the characteristics of earthquake motions and buildings, they are expressed as functions of many parameters to represent input motions to buildings or soil-structure systems.

The response spectra defined for the engineering bedrock are used as inputs to the soil-structure system, or the response spectra that consider amplification effect (including nonlinear characteristics) of surface soil and soil-structure interaction are used as inputs to the building.

The seismic loads that take into account the inelastic response of buildings during strong earthquake motions include the reduction factor related to ductility and response deformation that are defined the
7. SEISMIC LOADS Commentary

nonlinear characteristics of the building and its limit deformation. This reduction factor was given as a unique value for each direction of the building plan in the previous recommendations, but it is given for each story and in each direction in the current recommendations. Because it is possible to give the same value for all stories, the concept of the reduction factor is the same as in the previous recommendations.

For relatively regular buildings, a simplified method can be used to evaluate the equivalent static seismic loads, without performing eigenvalue analysis. In this case, it is also recommended that the natural period and damping ratio should be evaluated considering the soil-structure interaction.

For buildings irregular in plan or elevation or both, buildings with long spans, and very important buildings, dynamic analysis is performed using design earthquake motions as discussed in Sec.7.2, with additional recommendation that equivalent static analysis also be used.

Section 7.3 deals with design earthquake motions to be used for dynamic analysis, which were included in the commentary in the previous recommendations. This is because dynamic analysis has become prevalent in the design and technical development of evaluating design earthquake motions with reliable accuracy. There are two methods to evaluate design earthquake motions. The first method is to make design earthquake motions that fit the response spectrum, as in Sec.7.2. The second is to make design earthquake motions based on scenario earthquakes that take into account various conditions of the construction site. The first method makes design earthquake motions that fit the response spectrum at the engineering bedrock or at the location of input level of the building. The second method makes design earthquake motions based on the several scenario earthquakes, considering source effects, propagation path, and soil conditions.

Notations: Notations used in the text are as follows:

Roman upper-case notations

- $A_f$: area of the foundation base
- $G_a(\omega)$: acceleration power spectrum at the ground level or at the base of foundation
- $G_{a0}(\omega)$: acceleration power spectrum at the engineering bedrock
- $H(\omega)$: transfer function of acceleration
- $H_{GS}(\omega)$: soil amplification function to represent the amplification characteristics of surface soil
- $H_{SSI}(\omega)$: adjustment function to represent soil-structure interaction
- $Q_E$: live load for seismic load
- $S_0(T, \zeta)$: normalized acceleration response spectrum at the engineering bedrock
- $S_{a0}(T, \zeta)$: acceleration response spectrum at the engineering bedrock
- $S_a(T_j, \zeta_j)$: acceleration response spectrum at the ground level or at the base of foundation
- $T$: period
- $T_1$: fundamental natural period of the building
- $T_G$: predominant period of soil above the engineering bedrock
- $T_c, T_c'$: corner periods related to the predominant period of the engineering bedrock
- $T_d$: duration for spectral conversion
- $T_f$: fundamental natural period of the building fixed at the base
- $T_j$: $j$-th natural period of the building
- $T_r$: natural period of rocking assuming the building is rigid
7.1. Estimation of Seismic Loads

\( T_s \): natural period of sway assuming the building is rigid
\( V_{Ei} \): seismic shear of the \( i \)-th story of a building
\( V_{ij} \): seismic shear of the \( i \)-th story for the \( j \)-th natural mode
\( V_s \): shear wave velocity

**Roman lower-case notations**

\( a_0 \): basic peak acceleration at the engineering bedrock
\( g \): acceleration due to gravity
\( g(\eta, \omega) \): function to represent embedded foundation
\( i \): the story number or the imaginary unit
\( j \): maximum number of natural modes considered
\( k \): factor to represent statistical seismic hazard at the site
\( k_1, k_2 \): factors determined depending on the number of stories (or height or fundamental natural period \( T_1 \))
\( k_{D1} \): reduction factor related to ductility of the building
\( k_{F1} \): amplification factor due to the irregularities of the building
\( k_{R0} \): acceleration response ratio
\( k_{Vi} \): seismic shear distribution factor for the \( i \)-th story
\( k_p(T_j, \zeta_j) \): peak factor of the acceleration response of a single-degree-of-freedom system
\( k_{pE} \): return period conversion factor
\( n \): total number of stories of the building
\( r \): return period
\( r_{jk} \): natural frequency ratio of the \( j \)-th and \( k \)-th modes
\( w_i, w_k \): \( i \)-th, \( k \)-th weight

**Greek notations**

\( \alpha_G \): impedance ratio from the engineering bedrock to surface soil
\( \alpha_i \): normalized weight
\( \beta_j \): participation factor of the \( j \)-th natural mode
\( \delta_d \): normalized depth of the foundation
\( \zeta \): damping ratio
\( \zeta_1 \): damping ratio for the fundamental mode of the building
\( \zeta_G \): damping ratio of soil above the engineering bedrock
\( \zeta_r \): damping ratio for the fundamental mode of the building fixed at the base
\( \zeta_j, \zeta_k \): \( j \)-th and \( k \)-th mode damping ratios
\( \zeta_s \): damping ratio of sway assuming the building is rigid
\( \zeta_r \): damping ratio of rocking assuming the building is rigid
\( \eta \): ratio between embedment depth of foundation \( d \) and equivalent width \( l \)
\( \mu_m \): adjustment factor for single-degree-of-freedom assumption of multi-degree-of-freedom system
\( \rho_{jk} \): correlation factor for the \( j \)-th and \( k \)-th natural modes
\( \sigma_a(T_j, \zeta_j) \): root mean square of the acceleration response of a single-degree-of-freedom system
\( \phi_k \): \( j \)-th natural mode shape of the \( k \)-th story
\( \omega \): circular frequency
\( \omega_j \): \( j \)-th natural circular frequency of the building
7. SEISMIC LOADS Commentary

7.1.1 Seismic load and design earthquake motion

For ordinary buildings, an equivalent static load is calculated using a response spectrum method and is to be used for static stress analysis (this series of procedure may be referred to as the equivalent static analysis). The response spectrum method is basically applicable only for elastic structures, but can be used to approximately estimate elasto-plastic structures with uniform plasticity within the structures. Most often, the horizontal components of seismic loads are significant for ordinary buildings, the vertical components may be neglected. Sec.7.2 is written under the ordinary conditions mentioned above. In case that there is non-uniform plasticity within a building, or if the vertical vibration cannot be ignored, or for buildings which tend to behave as in the following items from a) to h), dynamic analyses (time-history response analyses) with design ground motions mentioned in Sec.7.3 should be implemented in order to verify the seismic safety.

a) a building with abrupt change in horizontal stiffness and strength in height and possible damage concentration in a particular story
b) a building with plane irregularity in mass and stiffness and significant torsional response
c) a building which is composed of frames with large-spanned beams and/or long cantilevers and the vertical vibration cannot be ignored
d) a non-multi-story and special shape building like shells and spatial structures
e) a tall building and a large sized building with great importance
f) a building which contains telecommunication devices and valuable contents, and whose structural dynamic response must be estimated
g) a building which contains dangerous materials, and whose failure may greatly affect the surroundings
h) a building with special devices such as base-isolation systems

Note that static analysis in Sec.7.2 is also preferably carried out in order to grasp the required safety level even in the case that dynamic analysis is implemented.

Although only a seismic load for static analyses described in Sec.7.2 is given in the previous version of the recommendations, the current recommendations cover the design ground motions for dynamic analyses described in Sec.7.3 of the main text.

It is described in Sec.7.2 that the seismic load used for the equivalent static analysis shall be calculated with the following two procedures. One procedure is a response spectrum method in which an eigenvalue analysis of a building model is first implemented and then superposition of each modal response based on the response spectrum is made to determine the seismic load. Another procedure is the simplified method without performing the eigenvalue analysis. In the above two procedures, the seismic load is based on the story-shear force computed from the acceleration response spectra $S_a(T_j, \zeta_j)$ and based on two modification factors: a reduction factor due to plastic deformation capacity and an amplification factor due to building irregularity.

The response spectrum $S_a(T_j, \zeta_j)$ is defined on the outcropped engineering bedrock and is based on the standard probabilistic seismic hazard map with respect to peak ground acceleration (PGA) taking account of spectral property, amplification of wave motion through shallow ground layers above the bedrock, and effect of soil-structure interaction.

Table 7.1.1 compares the seismic load of this recommendations and that of the previous version.

Once the response spectra is evaluated, the seismic load can be calculated as the design story shear force. When the vibrational modes higher than the first mode are significant, it is desirable to perform an eigenvalue analysis.
7.1. Estimation of Seismic Loads

### Table 7.1.1 Comparison of current and previous recommendations

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground motion intensity</td>
<td>( a_0 ): PGA on the engineering bedrock with ( V_S = 400 \text{ m/s} )</td>
<td>PGA on a standard outcropped ground (( V_i = 400 \sim 1000 \text{ m/s} )) based on a PGA map derived from an extreme value distribution</td>
</tr>
<tr>
<td>Wave amplification in surface layers</td>
<td>( H_{DG}(\omega) ): transfer function based on standard probabilistic seismic hazard map</td>
<td>( G_A, G_V ): soil modification factors</td>
</tr>
<tr>
<td>SSI (kinematic interaction)</td>
<td>( H_{SSI}(\omega) ): modification function representing embedment effect of a building</td>
<td>not considered</td>
</tr>
<tr>
<td>Response spectrum</td>
<td>( S_a(T, \zeta) )</td>
<td></td>
</tr>
</tbody>
</table>

### 7.1.2 Idealization of building and location of input ground motion

An actual building is very complex, but the building is often idealized into an adequate mathematical model to evaluate the seismic load. A lumped mass structural model is often used since most of mass is concentrated on each floor of the building. The building located on a firm ground can be modeled as a fixed base model as illustrated in Fig.7.1.1a. The mass concentration on the floor level is assumed to move horizontally and independently with disregard to the motion in another orthogonal direction. If the ground supporting a building is not stiff, the so called “SR model” (Sway-Rocking model), which can represent sway and rocking motions of the foundation, should be used to take account of the soil-structure interaction effect as shown in Fig.7.1.1b. A building model shown in Fig.7.1.1c may sometimes be used to take account of motions of the ground and pile foundations. Note that treatment of SSI effect is discussed in Sec.7.2.1(3).

If the mass and stiffness of a building is not evenly distributed, two orthogonal horizontal motions are not independent of each other and rotational (torsional) motion is excited. Therefore, three degrees of freedom (two horizontal and torsion) should be considered for each story. For such dynamic systems with torsional motion, response spectrum method can also be applied. But the assumption that the maximum of each mode appears independently may not be appropriate, and SRSS (square root of sum of squares) rule to estimate the maximum response cannot be used. In such cases, CQC (complete quadratic combination) rule is used considering the correlations between eigenmodes of vibration.
7. SEISMIC LOADS Commentary

Since the vertical motion of a building is usually small, it is often neglected. For buildings with long span beams or long span cantilever beams, not only horizontal motions but also vertical motion should be considered, and a dynamic model to take the vertical motion into account is often used.

Among analytical models, there is the frame model where the mass is allocated at each joint of structural members including restoring characteristics of the members, and a soil-foundation-structure model where the soil is treated as finite elements. In case the floor diaphragm is not stiff enough to assume the diaphragm to be rigid, a model which connects each rigid diaphragm by flexible members should be used.

It is well known that the soil conditions affect earthquake damage and many seismic design codes include soil profile types. In many cases, however, it is only to adjust seismic design forces or to modify the design spectra. In order to consider the dynamic characteristics of surface soil layers, it is necessary to model the soil layers to be analyzed. In this case, the earthquake motion is determined at the so-called engineering bedrock, and it is analyzed how the amplitude and frequency contents are affected through the soil layers. The earthquake motion is then determined at the ground surface or at the bottom of the foundation. The engineering bedrock is chosen as the stiff soil layer that is deeper than the layer supporting the foundation. The stiff soil layer for engineering purpose is the layer whose shear wave velocity is approximately more than 400m/s.

All the above analytical models should have appropriate damping. The damping ratio depends on the structural type and materials, and also on the stiffness of foundation and soil. The general feature of damping is indicated in the commentary of Sec.7.2.2(1).

7.2 Calculation of Seismic Loads

7.2.1 Methods for calculating seismic load

(1) Procedure with eigenvalue analysis

Room types in Table 7.1 are classified in Table 4.1 as follows:

① houses, dormitories, etc.
② bedrooms in hotels (excluding unit baths)
7.2. Calculation of Seismic Loads

- offices and laboratories
- department stores and shops
- computer rooms
- garages
- warehouses for books
- theaters, movie theaters, halls, assembly halls, meeting rooms, classrooms, etc.

1) Eigenvalue analysis

The equation of motion of a multi-degree-of-freedom (MDOF) system (Fig.7.2.1) subjected to the excitations such as earthquake motions at the base is given as follows.

\[
[m][\ddot{x}] + [c][\dot{x}] + [k][x] = -[m][e] \bar{x}_b
\]

(7.2.1)

where \([m], [c], [k]\) are the mass, damping and stiffness matrices, respectively, \(\{\ddot{x}\}, \{\dot{x}\}, \{x\}\) are the acceleration vector, velocity vector, displacement vector and \([e]\) is a vector where each element is one in case of fixed base.

The equation of motion for free vibration becomes as follows.

\[
[m][\ddot{x}] + [c][\dot{x}] + [k][x] = 0
\]

(7.2.2a)

The solution of the above equation can be assumed to be as follows.

\[
\{x\} = \{\phi\} e^{i\omega t}
\]

(7.2.3a)

The characteristic equation is given as follows.

\[
|\lambda^2[m] + \lambda[c] + [k]| = 0
\]

(7.2.4a)

Then eigenvalues are obtained as \(n\) pairs of complex number for \(n\)-degree of freedom. The real part and the imaginary part give \(j\)-th undamped natural frequency and damping ratio. From the eigenvalue \(\lambda_j\), \(j\)-th natural mode (it is called as eigenvector, natural mode of vibration, vibration mode or simply mode) is obtained and the total natural mode are also \(n\) pairs of vectors of complex numbers.

The above analysis (which is called complex eigenvalue analysis) is not easy to perform, the damping term of the equation is usually neglected and simplified as follows. That is, the undamped equation of motion of free vibration is given as follows.

\[
[m][\ddot{x}] + [k][x] = 0
\]

(7.2.2b)

The displacement vector \(\{x\}\) is assumed to be the product of the constant vector \(\{\phi\}\) and the function of time \(e^{i\omega t}\) as follows.

\[
\{x\} = \{\phi\} e^{i\omega t}
\]

(7.2.3b)
In order for the above equation to have nontrivial solution, the determinant \(|[k] - \omega^2[m]|\) must be zero.

\[
|k| - \omega^2|m| = 0
\]  \hspace{1cm} (7.2.4b)

Since this equation (called as eigenvalue equation or frequency equation) is an \(n\)-degree equation, \(n\) real eigenvalues are obtained and they give \(n\) natural frequencies \(\omega_j\) \((j = 1, 2, \cdots, n)\). Then, natural mode \(\{\phi_j\} \,(j = 1, 2, \cdots, n)\) can be calculated for each natural frequency.

Combining all natural modes, the mode matrix is given as follows.

\[
[\phi] = [[\phi_1]|\phi_2| \cdots |\phi_n]] = \begin{bmatrix}
\phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\
\phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\phi_{n1} & \phi_{n2} & \cdots & \phi_{nn}
\end{bmatrix}
\]  \hspace{1cm} (7.2.5)

Then the solution \(\{x\}\) of Eq.(7.2.1) is assumed to be the product of mode matrix and the function of generalized coordinate or the vector of the generalized coordinate of time.

\[
\{x\} = [\phi]|x^*| = [[\phi_1]|\phi_2| \cdots |\phi_n]]\begin{bmatrix} x_1^* \\ x_2^* \\ \vdots \\ x_n^* \end{bmatrix}
\]  \hspace{1cm} (7.2.6)

Then the equation of motion becomes as follows.

\[
[m][\phi]|x^*| + [c][\phi]|\ddot{x}^*| + [k][\phi]|x^*| = -[m]|e|\ddot{x}_g
\]  \hspace{1cm} (7.2.7)

Pre-multiplying the inverse matrix of \([\phi]\) to both sides of the above equation, we have

\[
[\phi]^T[m][\phi]|\ddot{x}^*| + [\phi]^T[c][\phi]|\dddot{x}^*| + [\phi]^T[k][\phi]|x^*| = -[\phi]^T[m]|e|\ddot{x}_g
\]  \hspace{1cm} (7.2.8)

orthogonality of natural mode, that is \(\{\phi_j\}^T[m]\phi_k = 0\), \(\{\phi_j\}^T[k]\phi_k = 0\) for \(j \neq k\), the next relationships are obtained.

\[
[\phi]^T[m][\phi] = \begin{bmatrix}
m_1^* & 0 & \cdots & 0 \\
0 & m_2^* & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & m_n^*
\end{bmatrix}
\]  \hspace{1cm} (7.2.9)

\[
[\phi]^T[k][\phi] = \begin{bmatrix}
k_1^* & 0 & \cdots & 0 \\
0 & k_2^* & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & k_n^*
\end{bmatrix}
\]  \hspace{1cm} (7.2.10)

where \(m_j^* = \{\phi_j\}^T[m]\phi_j\) is the \(j\)-th generalized mass and \(k_j^* = \{\phi_j\}^T[k]\phi_j\) is the \(j\)-th generalized stiffness.

The natural modes are not always orthogonal with respect to damping matrix, therefore the damping matrix generally becomes as follows.

\[
[\phi]^T[c][\phi] = \begin{bmatrix}
c_{11} & c_{12} & \cdots & c_{1n} \\
c_{21} & c_{22} & \cdots & c_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
c_{n1} & c_{n2} & \cdots & c_{nn}
\end{bmatrix}
\]  \hspace{1cm} (7.2.11a)
7.2. Calculation of Seismic Loads

In the analysis, it is assumed that orthogonality exists as Rayleigh damping, given by Eq. (7.2.18b), and this matrix is a diagonal matrix as follows.

\[
[\phi]^T [c] [\phi] = \begin{bmatrix}
c_1^* & 0 & \cdots & 0 \\
0 & c_2^* & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & c_n^*
\end{bmatrix}
\] (7.2.11b)

where \( c_j^* = \{\phi_j\}_1^T [c] [\phi_j] \) is the generalized damping.

Then the equation of motion becomes as follows.

\[
\begin{bmatrix}
m_1^* & 0 & \cdots & 0 \\
0 & m_2^* & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & m_n^*
\end{bmatrix}
\begin{bmatrix}
x_1^* \\
x_2^* \\
\vdots \\
x_n^*
\end{bmatrix} + \begin{bmatrix}
c_1^* & 0 & \cdots & 0 \\
0 & c_2^* & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & c_n^*
\end{bmatrix}
\begin{bmatrix}
x_1^* \\
x_2^* \\
\vdots \\
x_n^*
\end{bmatrix} = \begin{bmatrix}
k_1^* & 0 & \cdots & 0 \\
0 & k_2^* & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & k_n^*
\end{bmatrix}
\begin{bmatrix}
x_1^* \\
x_2^* \\
\vdots \\
x_n^*
\end{bmatrix}
\]

This matrix equation is the same as uncoupled differential equations as follows.

\[
m_1^* \ddot{x}_1^* + c_1^* \dot{x}_1^* + k_1^* x_1^* = -\ddot{x}_g \sum_{i=1}^{n} m_i \phi_{i1} \\
m_2^* \ddot{x}_2^* + c_2^* \dot{x}_2^* + k_2^* x_2^* = -\ddot{x}_g \sum_{i=1}^{n} m_i \phi_{i2} \\
\vdots \\
m_n^* \ddot{x}_n^* + c_n^* \dot{x}_n^* + k_n^* x_n^* = -\ddot{x}_g \sum_{i=1}^{n} m_i \phi_{in}
\] (7.2.12b)

or

\[
\begin{bmatrix}
\ddot{x}_1^* + 2 \zeta_1 \omega_1 \dot{x}_1^* + \omega_1^2 x_1^* = -\beta_1 \ddot{x}_g \\
\ddot{x}_2^* + 2 \zeta_2 \omega_2 \dot{x}_2^* + \omega_2^2 x_2^* = -\beta_2 \ddot{x}_g \\
\vdots \\
\ddot{x}_n^* + 2 \zeta_n \omega_n \dot{x}_n^* + \omega_n^2 x_n^* = -\beta_n \ddot{x}_g
\end{bmatrix}
\] (7.2.12c)

where

\[
\omega_j = \sqrt{\frac{k_j^*}{m_j}} \quad 2 \zeta_j \omega_j = \frac{c_j^*}{m_j} \quad \beta_j = \frac{\sum_{i=1}^{n} m_i \phi_{ij}}{\sum_{i=1}^{n} m_i \phi_{ij}^2}
\] (7.2.13 a, b, c)

Each equation of Eq. (7.2.12c) is the equation of motion of a SDOF system with natural circular frequency \( \omega_j \) and damping ratio \( \zeta_j \) for \( j = 1, 2, \ldots, n \) subjected to the input motion multiplied by the participation factor \( \beta_j \).

The solution \( \{x\} \) is obtained using the relationship \( \{x\} = [\phi]\{x^*\} \) of Eq. (7.2.6). That is

\[
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_n
\end{bmatrix} = \begin{bmatrix}
\phi_{11} & \phi_{12} & \cdots & \phi_{1n} \\
\phi_{21} & \phi_{22} & \cdots & \phi_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\phi_{n1} & \phi_{n2} & \cdots & \phi_{nn}
\end{bmatrix}
\begin{bmatrix}
x_1^* \\
x_2^* \\
\vdots \\
x_n^*
\end{bmatrix} = \begin{bmatrix}
\phi_{11} x_1^* + \phi_{12} x_2^* + \cdots + \phi_{1n} x_n^* \\
\phi_{21} x_1^* + \phi_{22} x_2^* + \cdots + \phi_{2n} x_n^* \\
\vdots \\
\phi_{n1} x_1^* + \phi_{n2} x_2^* + \cdots + \phi_{nn} x_n^*
\end{bmatrix}
\] (7.2.14)

In this manner, the response for each natural mode is calculated independently, and the total response is obtained as the sum of these responses. This method of analysis is called modal analysis or mode superposition method. This method gives exact solution for linear systems if all modes are superimposed. It is also used as an approximate method when superimposing the important modes only.
2) Response spectrum method

The maximum response for each mode can be obtained from the response spectrum, i.e. the response spectra of displacement \( S_d(T_j, \zeta_j) \), velocity \( S_v(T_j, \zeta_j) \) and acceleration \( S_a(T_j, \zeta_j) \) give maximum responses of displacement, velocity and acceleration of the \( j \)-th mode and \( i \)-th story, and are determined as follows.

\[
\begin{align*}
    x_{ij} &= \beta_j \phi_{ij} S_d(T_j, \zeta_j) \\
    \dot{x}_{ij} &= \beta_j \phi_{ij} S_v(T_j, \zeta_j) \\
    \ddot{x}_{ij} &= \beta_j \phi_{ij} S_a(T_j, \zeta_j)
\end{align*}
\]  

(7.2.15)

The maximum lateral force \( p_{ij} \) of the \( i \)-th story and the \( j \)-th mode is calculated as the acceleration value of Eq.(7.1.15) multiplied by the mass \( m_i = w_i/g \).

\[
p_{ij} = \beta_j \phi_{ij} S_a(T_j, \zeta_j) \frac{w_i}{g} \tag{7.2.16}
\]

The maximum shear force of the \( i \)-th story and the \( j \)-th mode is obtained as the sum from the top to the \( i \)-th story and the \( j \)-th mode as follows. That is the maximum shear force of the \( i \)-th story and the \( j \)-th natural mode is given as Eq.(7.2) in the text as follows.

\[
V_{ij} = \frac{S_a(T_j, \zeta_j)}{g} \sum_{k=i}^{n} (w_k \beta_j \phi_{kj}) \tag{7.2}
\]

When the maximum total response is evaluated from the maximum response of each mode, the sum of the maximum response of each mode overestimates the maximum total response, since the maximum response for each mode does not occur simultaneously. Therefore the maximum total response is evaluated as the square root of sum of squares (SRSS) of the maximum response for each mode. Then the maximum story shear force is evaluated as follows.

\[
V_i = \sqrt{\sum_{j=1}^{n} V_{ij}^2} \tag{7.2.17}
\]

Since the fundamental mode is the most influential and the higher mode is less influential in the above equation, the sum need not be from the fundamental mode to the highest mode. This recommendation proposes the sum from the fundamental mode to the \( j_c \)-th mode, but the sum up to third mode is satisfactory.

The reduction factor \( k_{Di} \) related to ductility of the building and the amplification factor according to the irregularities of the building \( k_{Fi} \) are then multiplied, and the seismic loads as story shear forces are evaluated as Eq.(7.1) in the text as follows.

\[
V_{Ei} = k_{Di} k_{Fi} \sqrt{\sum_{j=1}^{j_c} V_{ij}^2} \tag{7.1}
\]

For buildings with less irregularities in the distributions of stiffness and mass in plan, the maximum total response can be evaluated using the SRSS rule. But torsional vibration cannot be neglected for buildings with larger eccentricities, and the probability is high that eigenvalues for different mode are close to each other. Therefore, when it is estimated that the different mode have closer eigenvalues, instead of using Eq.(7.1) \( V_{Ei} \) is calculated as follows.

\[
V_{Ei} = k_{Di} k_{Fi} \sqrt{\sum_{j=1}^{j_c} \sum_{k=1}^{j_c} \rho_{jk} V_{ik}} \tag{7.3}
\]
where the correlation factor $\rho_{jk}$ between the $j$-th and the $k$-th natural modes is given as follows.

$$\rho_{jk} = \frac{8 \sqrt{\zeta_j \zeta_k (\zeta_j + r_{jk} \zeta_k)^3}{(1 - r_{jk})^2 + 4 \zeta_j \zeta_k (1 + r_{jk}^2) + 4 (\zeta_j^2 + \zeta_k^2) r_{jk}^2}}$$  \hspace{1cm} (7.4)

The $r_{jk}$ is the ratio of the $j$-th and the $k$-th natural frequencies ($r_{jk} = \omega_j / \omega_k = T_k / T_j$). Eq.(7.4) is derived from the random vibration theory and it is called complete quadratic combination (CQC) rule$^{3,4}$. In this case, the $j_c$ should be large enough so that all influential modes are included to get the sum.

3) Damping

The phenomenon that the amplitude of vibration decreases gradually as time goes on is called “damping” and its degree is usually presented by damping ratio. The more damping the structure has, the less response the structure experiences when it is subjected to excitations such as earthquake motions.

The damping within the building is caused by transformation into thermal energy caused by the friction of connections and deformation of members, energy consumption caused by inelastic deformation of members and connections in large deformation, etc. The damping in the ground is caused by soil viscosity, dispersion of wave energy to adjacent ground, inelastic hysteresis, etc.

The cause of damping is classified as shown in Fig.7.2.2. The distinction between internal and external damping is whether the cause of damping is within the dynamic system or not.

![Fig.7.2.2 Classification of damping](image)

The frequency dependency of the damping ratio depends on the model considered. The damping ratio is proportional to the frequency for the Voigt model where spring and dashpot are in series. The damping ratio is inversely proportional to the frequency for the Maxwell model where spring and dashpot are in parallel. The hysteretic damping does not depend on the frequency and is constant$^5$.

In dynamic analysis of buildings, damping coefficient that is proportional to stiffness given by Eq.(7.2.18a) is frequently used. In this case, damping ratio becomes proportional to frequency and it gives higher damping for higher modes. The damping coefficient of Rayleigh type is the combination
of mass proportional damping and stiffness proportional damping as given by Eq.(7.2.18b). In this case, the damping ratio is the combination of mass proportional damping and frequency proportional damping.

\[
[c] = a_1[k]
\]

(7.2.18a)

\[
[c] = a_0[m] + a_1[k]
\]

(7.2.18b)

where \(a_0\) and \(a_1\) are constants.

It is reported that damping ratios for higher modes do not become much larger than for fundamental mode through experimental records\(^6\). Therefore, damping ratios can be underestimated for high-rise buildings, if the damping coefficient is assumed to be stiffness proportional. Experimental records for high-rise buildings show that the higher the building, the smaller is the damping ratio, and the larger the amplitude, the larger the damping ratio. It is also reported that the larger the plan dimension, the larger the variation of the damping ratio\(^7\).

Design damping ratios for different structures are as follows.

\[
\begin{align*}
\text{Reinforced concrete} &: \quad \zeta = 0.02 \sim 0.04 \\
\text{Steel} &: \quad \zeta = 0.01 \sim 0.03
\end{align*}
\]

(7.2.19)

These values include the internal damping of the structure and do not include radiation damping to the ground. The values also depend on strain level and type of structures.

Response spectra for seismic loads are often indicated for the 5% damping ratio\(^8\). Several formulae are proposed to obtain the response spectra for different damping ratios. The response spectra for the 5% damping ratio may be modified as follows\(^9, 10\).

\[
\mu_d = \frac{1.5}{1 + 10\zeta}
\]

(7.2.20)

When different damping ratios are given for parts or members of the system, the damping ratio for each mode is rigorously calculated through the complex eigenvalue analysis. But the damping ratios can be approximated applying the weighted modal damping concept\(^11\), where damping ratios for different parts are weighted depending on the strain mode ratios.

### 4) Live load for seismic load

The live load for seismic load is the weight of objects per unit floor area that are fixed to the building. Since the motion of people inside the building differs from the building, live load caused by people is not included for seismic load. Only the objects fixed to the building are included, since they move along with the building, and the objects that may slide in case of earthquakes are not included. But it is difficult to decide whether it is fixed or not for all objects in the building. Therefore, average weights of fixed objects for the room types in Table 4.3 are shown in Table 7.1, that can be used as the live load for seismic load. Background data for Table 7.1 are shown in Sec.4.3.2.

An example to calculate the object weight is shown in Fig.4.5.1, where the tributary area is the influence area of the floor. The objects considered are the same as in Sec.4.2. Therefore, if there are some objects that are not considered in Sec.4.2, the load in Table 7.1 should be increased. In case live load consists mainly of people, the live load for seismic load is set to be 300 N/m\(^2\) considering fixed chairs, furniture, etc. Each value in the table has been decided looking into Table 4.2.5. The weight of fixed chairs was obtained through simulation using the data of weight listed in the catalogue.
7.2. Calculation of Seismic Loads

Takanashi, et al.\textsuperscript{12} showed that equivalent acceleration response can be reduced in short period range in case objects in the building slide during earthquakes. In order to consider the effect of sliding objects in seismic design, the method is proposed to use the equivalent live load where the weight of sliding objects is excluded. In the long period range, objects may not slide, but the live load caused by people can be excluded. Therefore, the method to reduce the live load for seismic load can be used in all period range.

From the above discussions, in case the objects slide during earthquakes, the live load may be reduced considering the situations and conditions of the objects.

(2) Procedure without eigenvalue analysis

For regular buildings in terms of plan and elevation, a simplified method to calculate seismic loads may be used in order to avoid performing eigenvalue analysis. Then the seismic loads are approximately calculated, estimating or calculating the fundamental natural period of the building, and using a formula for the seismic shear distribution factor of the story, since it has been proven that the simplified method may not endanger the seismic safety of regular buildings. In order to understand the background of such concept, analytical examples for continuous bodies and lumped mass systems are shown.

1) Shear forces of uniform shear body

Buildings are often idealized as a uniform continuum. The analysis for a uniform shear body fixed at the base is performed as follows\textsuperscript{13, 14}.

The \( j \)-th seismic shear force \( V_{ij} \) at the level \( i \) is calculated as the sum of lateral forces from the top to that level, and it is given replacing the sum of Eq.(7.3) by the integral as follows.

\[
V_{ij} = \frac{S_{aj}}{g} W \int_0^{\alpha_i} \beta_j \phi_j(\alpha) \, d\alpha \quad (7.2.21)
\]

where \( S_{aj} \) is the \( j \)-th acceleration response, \( W \) is the total weight of the building, \( \beta_j \) is the \( j \)-th participation factor, \( \phi_j(\alpha) \) is the \( j \)-th natural mode, and \( \alpha \) is the normalized weight which is given as the weight from the top to the level considered divided by the total weight.

For uniform shear body, \( \phi_j(\alpha) \) and \( \beta_j \) are given as follows.

\[
\phi_j(\alpha) = \cos \left( \frac{(2j-1)\pi}{2} \alpha \right) \quad (7.2.22)
\]

\[
\beta_j = \frac{(-1)^{j+1} 4}{(2j-1)\pi} \quad (7.2.23)
\]

Substituting Eqs.(7.2.22) and (7.2.23) into Eq.(7.2.21), we have

\[
V_{ij} = \frac{(-1)^{j+1} 8 W S_{aj}/g}{(2j-1)\pi^2} \sin \left( \frac{(2j-1)\pi}{2} \alpha_i \right) \quad (7.2.24)
\]

Assuming that maximum response of different modes do not occur simultaneously, the maximum total response is estimated using SRSS rule as follows.

\[
V_i = 8 W \sqrt{\sum_{j=1}^{\infty} \frac{(S_{aj}/g)^2}{(2j-1)^4 \pi^4} \sin^2 \left( \frac{(2j-1)\pi}{2} \alpha_i \right)} \quad (7.2.25a)
\]
In order to evaluate the above equation, the acceleration response for each mode should be given, and it is assumed to be as follows.

\[ S_{aj} = S_{a1} \left( \frac{\omega_j}{\omega_1} \right)^k \]  

(7.2.26a)

where \( \omega_j \) is the natural frequency of the \( j \)-th mode.

For uniform shear body fixed at the base,

\[ \omega_j = (2j-1) \omega_1 \]  

(7.2.27)

Therefore, Eq.(7.2.26a) becomes as follows.

\[ S_{aj} = (2j-1)^k S_{a1} \]  

(7.2.26b)

Substituting Eq.(7.2.26b) into Eq.(7.2.25a), we have

\[ V_i = \frac{8}{\pi^2} \frac{S_{a1}}{g} W \sqrt{\sum_{j=1}^{\infty} \frac{1}{(2j-1)^{4-2k}} \sin^2 \left( \frac{(2j-1)\pi}{2} \alpha_i \right)} \]  

(7.2.25b)

Acceleration response is considered to be almost constant (see the horizontal line of \( S_a \) constant in Fig.7.2.4) for short period or low-rise buildings, i.e. \( k = 0 \) in Eq.(7.2.26b), and Eq.(7.2.25b) becomes as follows.

\[ V_i = \frac{8}{\pi^2} \frac{S_{a1}}{g} W \sqrt{\frac{3\pi^4 \alpha_i^2 - 2\pi^4 \alpha_i^3}{96}} = \sqrt{\frac{2}{3}} \frac{S_{a1}}{g} W \alpha_i \sqrt{3 - 2\alpha_i} \]  

(7.2.28a)

Substituting \( \alpha_i = 1 \) into Eq.(7.2.28a), the base shear force \( V_B \) of the fixed base is given as follows.

\[ V_B = \sqrt{\frac{2}{3}} \frac{S_{a1}}{g} W \approx 0.816 \frac{S_{a1}}{g} W \]  

(7.2.28b)

This equation indicates that the base shear force of the uniform shear body fixed at the base for the constant acceleration response is 0.816 of the SDOF system that has the same natural period.

Velocity response is considered to be almost constant (see the curve of \( S_v \) constant in Fig.7.2.4) for long period or high-rise buildings, i.e. \( k = 1 \) in Eq.(7.2.26b), and Eq.(7.2.25b) becomes as follows.

\[ V_i = \frac{8}{\pi^2} \frac{S_{a1}}{g} W \sqrt{\frac{\pi^2}{8} \alpha_i} = \frac{2 \sqrt{2}}{\pi} \frac{S_{a1}}{g} W \sqrt{\alpha_i} \]  

(7.2.29a)

Then the base shear force becomes as follows.

\[ V_B = \frac{2 \sqrt{2}}{\pi} \frac{S_{a1}}{g} W \approx 0.900 \frac{S_{a1}}{g} W \]  

(7.2.29b)

This equation indicates that the base shear force of the uniform shear body fixed at the base for the constant velocity response is 0.900 of the SDOF system that has the same natural period.

The adjustment factor for single-degree-of-freedom assumption of multi-degree-of-freedom system in Eq.(7.5) is 0.82 for short-period buildings as shown in Eq.(7.28b) and 0.90 for long-period buildings as indicated in Eq.(7.2.29b).

In case a multi-story building has a soft-first-story, \( \mu_m = 1.0 \), because only the fundamental mode becomes predominant. Therefore, seismic loads as story shear forces can not be decreased by using this factor for soft-first-story buildings.
7.2. Calculation of Seismic Loads

Fig. 7.2.5a shows the distributions of seismic shear force $V_i/V_B$ given by Eq. (7.2.28a) normalized by the base shear, its seismic shear coefficient $k_{Vi}$ and its seismic coefficient $k_{pi}$ normalized by the base shear coefficient. The distributions of seismic shear coefficient and seismic coefficient caused by the constant acceleration response spectrum are almost linearly increasing from the bottom to the top and they are similar to Fig. 7.2.5b for inverted triangular distribution of seismic coefficient. The distribution of seismic shear is a parabola where the vertex is at the bottom.

Fig. 7.2.5c indicates the distributions of $V_i/V_B$, $k_{Vi}$ and $k_{pi}$ given by Eq. (7.2.29a). The seismic shear coefficient and seismic coefficient become infinitely large at the top. The seismic shear force, however, is not infinitely large but converges to zero at the top. The distribution of seismic shear is also a parabola, but the vertex is at the top.

Fig. 7.2.5d indicates the distributions of $V_i/V_B$, $k_{Vi}$ and $k_{pi}$ for uniform distribution of seismic coefficient.

2) Shear forces of lumped mass systems

Since the continuum cannot always represent real buildings and also it is difficult to analyze the non-uniform continuum, lumped mass models are often used to analyze buildings. Some analytical
Ishiyama analyzed shear, flexural and shear-flexural 5DOF models varying the distributions of mass and stiffness, then proposed the formula to give appropriate distribution of seismic shear forces as follows:\(^{15}\)

The distribution of seismic shear force is represented by the combination of four distributions as follows.

a) Uniform distribution of seismic coefficient \(V_a\) in Fig.7.2.6

b) Inverted triangular distribution of seismic coefficient \(V_b\) in Fig.7.2.6

c) Distribution of shear type structure subjected to white noise or constant velocity response spectrum (this may be called \(\sqrt{\alpha}\) distribution, since the distribution of seismic shear force is proportional to \(\sqrt{\alpha}\)) \(V_c\) in Fig.7.2.6

d) Distribution of higher mode effect of flexural structure \(V_d\) in Fig.7.2.6

The distribution of seismic shear force is expressed as the combination of above four distributions, \(V_a, V_b, V_c\) and \(V_d\), all normalized by the base shear, as follows.

\[
\frac{V_i}{V_B} = V_a + k_1(V_b - V_a) + k_2(V_c - V_a) + k_3(V_d - V_a) \quad (7.2.30)
\]

where \(k_1, k_2\) and \(k_3\) are the factors that depend on the characteristics of the building and earthquake motions.

The above formula normalized by the base shear coefficient is given as follows.

\[
k_{V_i} = \frac{k_i}{k_{VB}} = 1 + k_1(1 - \alpha_i) + k_2(\frac{1}{\sqrt{\alpha_i}} - 1) + k_3(0.2 - \alpha_i)(1 - \sqrt{\alpha_i})^2 \quad (7.2.31)
\]

The first three terms of the right side are derived analytically by a), b) and c). The last term is derived empirically by d). Since the value of Eq.(7.2.31) is so small, that the distribution for \(k_3 = 10\) is shown in Fig.7.2.6.

Analyzing various models, the three factors to give appropriate distribution have been proposed as follows.

\[
\begin{align*}
k_1 &= \frac{0.05}{0.05 + r} \frac{s^2}{0.5 + s^2} \frac{4}{4 + r^2} + \frac{2}{3} \frac{r}{0.05 + r} \frac{1.5 + s^2 + t^2}{1 + s^2 + t^2} \\
k_2 &= \frac{0.05}{0.05 + r} \frac{s}{0.2 + s^2} \frac{t^2}{4 + r^2} + \frac{2}{3} \frac{r}{0.05 + r} \frac{s^2 + t^2}{1 + s^2 + t^2} \quad (7.2.32) \\
k_3 &= \frac{r}{0.2 + r} \frac{30r^2}{0.1 + s} \frac{s}{9 + r^2}
\end{align*}
\]

where \(r\) is the ratio of shear deformation to flexural deformation, \(s\) is lateral stiffness of the first story to the average stiffness of the building, and \(t\) is the ratio of the fundamental period of the building to the corner period of the spectrum. For the \(n\)-story building, equivalent stiffness ratio \(s_e = s^{5/n}\) is used instead of \(s\).
7.2. Calculation of Seismic Loads

Except for very slender high-rise buildings, buildings can be represented by shear type model. Substituting $r = 0$ that represent shear type model, the above equation becomes as follows.

$$k_{V_i} = \frac{k_i}{k_{VB}} = 1 + \frac{s^2}{0.5 + s^2} \frac{4}{4 + t^2}(1 - \alpha_i) + \frac{t^2}{0.2 + s^2} \frac{1}{4 + t^2}(\frac{1}{\sqrt{\alpha_i}} - 1) \quad (7.2.33)$$

The above equation was included in the comments of previous recommendations (1993 edition). In the current recommendations, $k_{V_i}$ to represent seismic shear force distribution is expressed by Eq.(7.2.31) eliminating the last term that expresses the distribution of higher mode effect in flexural structure.

$$k_{V_i} = 1 + k_1(1 - \alpha_i) + k_2\left(\frac{1}{\sqrt{\alpha_i}} - 1\right) \quad (7.6)$$

The values of factors $k_1$ and $k_2$ depend on the building height or the number of stories; $k_1 \approx 1$ and $k_2 \approx 0$ for low-rise buildings, $k_1 \approx 0$ and $k_2 \approx 1$ for high-rise buildings, and intermediate values for other buildings. The engineer can determine these actual values.

Letting $k_1 = k_2 = 2T/(1 + 3T)$ in Eq.(7.6) gives the so-called $A_i$ distribution of the Building Standard Law of Japan\(^{16}\). This may be referred to determine the values of $k_1$ and $k_2$.

$$k_{V_i} = A_i = 1 + \left(\frac{1}{\sqrt{\alpha_i}} - \alpha_i\right) \frac{2T}{1 + 3T} \quad (7.2.34)$$

3) Natural period

As already mentioned in Sec.7.2.1 (Procedure with eigenvalue analysis), the natural period and the corresponding natural mode shape can be obtained through eigenvalue analysis of the equation of motion of the structure. Besides rigorous method as eigenvalue analysis, the natural period can be estimated through simplified method. Several formulae have been proposed to approximately estimate the fundamental natural period, as follows.

a) Weight method

Let the maximum displacement of the structure (e.g. at the top of the structure) become $\delta$ (cm) when applying the inertia forces caused by the gravity in the direction concerned, the fundamental natural period $T_1$ (s) of the structure is expressed as follows.

$$T_1 = \frac{\sqrt{\delta}}{5} \sim \frac{\sqrt{\delta}}{5.7} \quad (7.2.35)$$

$\sqrt{\delta}/5$ is used for SDOF structure and $\sqrt{\delta}/5.7$ is for MDOF structure. The larger denominator indicates that the fundamental mode shape becomes larger at the top (free end) than the uniform mode shape.

b) Approximation of natural periods for buildings

Through investigations of existing buildings, regression formulae have been proposed to estimate the fundamental natural period $T_1$ (s) from the height $H$ (m) or the number of stories $n$. These formulae are convenient to be used, since the fundamental period can be approximately obtained without making the analytical model of the building.

For standard shear type structures, there is a formula to estimate the fundamental natural period in terms of the height $h$ (m) as follows\(^{17}\).

$$T_1 = (0.02 + 0.01\alpha_h)h \quad (7.2.36)$$
where \( \alpha \) is the ratio of steel structure to the total height of the building.

There is another formula to estimate the fundamental natural period in terms of the number of stories \( n \) as follows\(^{(18)}\):

\[
\begin{align*}
\text{For reinforced concrete structures:} \quad & T_1 = (0.06 \pm 0.02) n \\
\text{For steel structures:} \quad & T_1 = (0.1 \pm 0.03) n
\end{align*}
\]

(7.2.37)

Since the fundamental natural period varies depending on the amount of walls and/or braces and on the strain level, the above formulae have some margin.

(3) Consideration of soil-structure interaction

1) Seismic load considering soil-structure interaction

When a building is located on a firm ground, it is appropriate to assume that the ground is rigid. However in the case that the assumption is not appropriate, it is necessary to apply an analytical model considering the deformation of soil. The model is called the sway-rocking model (SR model)\(^{(2)}\) considering horizontal motion (sway) and rotational motion about horizontal axis (rocking) due to soil deformation. The equations of motion for this model are similar to Eq.(7.2.1).

![Fig.7.2.7 SR model](image1)

![Fig.7.2.8 Displacements of SR model](image2)

The model for super structure is the same as in Fig.7.2.1, the mass and rotational inertia of the foundation are \( m_0, I_0 \), the stiffness and damping coefficient for sway are \( k_s \) and \( c_s \) and those for rocking are \( k_r \) and \( c_r \). The mass matrix, displacement and \([e]\) vectors are given below. Since only the horizontal ground motion is considered and the rotational ground motion is not considered, the component for rocking in vector \([e]\) of Eq.(7.2.1) is 0.
7.2. Calculation of Seismic Loads

An MDOF model of a superstructure is often applied considering the equivalent single degree of freedom model (SDOF) of the superstructure or Eq.(7.2.42) using natural modes. The seismic shear force is calculated by Eq.(7.3).

The damping ratio is obtained by Eq.(7.2.13b) using appropriate damping coefficients. In the eigenvalue analysis, there are two methods; one is complex analysis including damping matrix and the other is performed without damping. The former analysis is less applied because eigenvalues become complex numbers so its use is complicated. The analysis is generally performed by the latter method. The damping ratio is obtained by Eq.(7.2.13b) or Eq.(7.2.42) using natural modes. The seismic shear force is calculated by Eq.(7.3).

The equivalent single degree of freedom model (SDOF) of the MDOF model of a superstructure is often applied considering sway and rocking motions and is called condensed SR model (Fig.7.2.9). The mass, stiffness and damping ratio of the SDOF model are \( m_e \), \( k_e \) and \( c_e \), respectively. The total stiffness, \( k_t \) is related to the stiffness of each element in Fig.7.2.9 as follows.

\[
\frac{1}{k_t} = \frac{1}{k_e} + \frac{1}{k_s} + \frac{h_e^2}{k_e} \tag{7.2.40}
\]

Multiplying \( m_e \) and using \( T = 2\pi \sqrt{m/k} \), Eq.(7.2.40) is transformed as follows.

\[
T_t^2 = T_e^2 + T_s^2 + T_r^2 \tag{7.2.41}
\]

As \( T_e \) is equal to \( T_t \) and \( T_t \) is the first natural period \( T_1 \) of the model in Fig.7.2.9, Eq.(7.8) in the text is obtained.

\[
T_1 = \sqrt{T_t^2 + T_s^2 + T_r^2} \tag{7.8}
\]

It is possible to apply Eq.(7.8) in either cases where \( T_t \) is given by an appropriate formula without eigenvalue analysis or it is obtained from the eigenvalue analysis.
In order to obtain the damping ratio of the condensed SR model, let us consider the strain and absorbed energy $E$ and $\Delta E$ during a cycle vibrating in the period $T_1$. The damping ratio is expressed as follows$^6$.

$$\zeta = \frac{1}{4\pi} \frac{\Delta E}{E} \quad (7.2.42)$$

The total displacement is as follows.

$$x_t = x_e + x_s + \theta h_e \quad (7.2.43)$$

The strain energy is as follows.

$$E = \frac{1}{2} (k_e x_e^2 + k_s x_s^2 + c \theta^2) \quad (7.2.44)$$

The absorbed energy is as follows.

$$\Delta E = \pi \omega_1 c_t x_t^2 = \pi \omega_1 (c_e x_e^2 + c_s x_s^2 + c \theta^2) \quad (7.2.45)$$

Substituting those into Eq.(7.2.42), the total damping ratio $\zeta_t$ becomes

$$\zeta_t = \frac{\pi \omega_1 (c_e x_e^2 + c_s x_s^2 + c \theta^2)}{2 \pi k_t x_t^2} \quad (7.2.46)$$

Using the following relationship,

$$\zeta_e = \frac{c_e}{2 \sqrt{m_e k_e}} \quad \zeta_s = \frac{c_s}{2 \sqrt{m_s k_s}} \quad \zeta_r = \frac{c_r}{2 \sqrt{m_r h_e^2 k_r}} \quad (7.2.47a, b, c)$$

$$\frac{x_e}{x_t} = \frac{k_e}{k_t} = \left( \frac{T_e}{T_1} \right)^2 \quad \frac{x_s}{x_t} = \frac{k_s}{k_t} = \left( \frac{T_s}{T_1} \right)^2 \quad \frac{x_r}{x_t} = \frac{h_e^2 k_r}{k_t} = \left( \frac{T_r}{T_1} \right)^2 \quad (7.2.48a, b, c)$$

The damping ratio $\zeta_1$ for the first mode in the SR model is calculated as Eq.(7.9) in the text as follows.

$$\zeta_1 = \zeta_t \left( \frac{T_1}{T_1} \right)^3 + \zeta_s \left( \frac{T_s}{T_1} \right)^3 + \zeta_r \left( \frac{T_r}{T_1} \right)^3 \quad (7.9)$$

Considering the condensed SR model to be SDOF, the shear force at each story $V_i$ of a building to be MDOF is obtained from Eq.(7.2.17) as follows.

$$V_i = \sqrt{V_{i1}^2 + \Delta V_i^2} \quad (7.2.49)$$

where $\Delta V_i$ is an additional shear force considering the higher modes.

The story shear force of the first mode is obtained as follows.

$$V_{i1} = \frac{S_a(T_1, \zeta_1)}{g} \sum_{k=1}^{n} (w_k \beta_{i1} \phi_{i1}) \quad (7.2.50)$$

The $T_1$ and $\zeta_1$ are obtained from Eqs.(7.8) and (7.9), respectively. The fundamental natural mode is approximated as follows.

$$\phi_{i1} = \phi_{i1} + \phi_s + \phi_r \frac{H_k}{h_e} \quad \phi_s = \frac{1}{\beta_{i1}} \left( \frac{T_s}{T_1} \right)^2 \quad \phi_r = \frac{1}{\beta_{i1}} \left( \frac{T_r}{T_1} \right)^2 \quad (7.2.51a, b, c)$$

where the $\phi_{i1}$ and $\beta_{i1}$ are natural modes and participation factor at the 1st mode under fixed condition of the superstructure. The factor $\beta_{i1}$ in Eq.(7.2.50) is obtained from Eq.(7.2.13c) using the modes in Eq.(7.2.51).
7.2. Calculation of Seismic Loads

The shear force \( \Delta V_i \) in Eq.(7.2.49) is obtained as follows.

\[
\Delta V_i = \frac{S_a(T_2, \zeta_2)}{g} \left\{ \sum_{k=1}^{n} w_k - \sum_{k=1}^{n} (w_k \beta_1 \phi_{k1}) \right\}
\]  \hspace{1cm} (7.2.52)

where \( T_2 \) and \( \zeta_2 \) are natural period and damping ratio of the second mode of SR model expressing in the similar manner as the first which are obtained using the eigenvalues of the second mode of superstructure with fixed base.

2) Stiffness and damping for sway and rocking

For flat, embedded and pile foundations the stiffness, \( K \), and damping coefficient, \( C \), for sway and rocking motions are obtained from the relationship between force and displacement at the base of the superstructure\(^\text{21}\). When a foundation is subjected to a force, \( P = P_0 e^{i \omega t} \), the displacement \( U = U_0 e^{i \omega t} \) is obtained, and the complex impedance function, \( K_d(\omega) \) is given as follows.

\[
K_d(\omega) = \frac{P}{U} = K(\omega) + i K'(\omega)
\]  \hspace{1cm} (7.2.53)

where \( K(\omega) \) and \( K'(\omega) \) are the real and imaginary parts of an impedance function, respectively.

The impedance function of a foundation in multi-layered soil with rigidity and damping become an irregular function in terms of frequency as dotted lines in Fig.7.2.10. When the effects of sway and rocking are expressed by a spring with stiffness, \( k \), and a dashpot with damping coefficient, \( c \), as in Fig.7.2.7 or Fig.7.2.9, the stiffness and damping coefficient are obtained as follows.

\[
k = K(0) \quad \text{(7.2.54a)}
\]

\[
c = \frac{K'(\omega_1)}{\omega_1} \quad \text{(7.2.54b)}
\]

where \( \omega_1 \) is the fundamental natural circular frequency of the interaction model. The definitions in Eq.(7.2.54) indicate that \( k \) is calculated statically and \( c \) is determined as the damped force by a dashpot at \( \omega_1 \).

3) Calculation of stiffness and damping for sway and rocking

The impedance function (or the stiffness and damping) of sway and rocking are evaluated applying the method based on a) the wave propagation theory, b) finite element method and c) discrete spring type method\(^\text{21}\). They have corresponding characteristics that a) wave propagation in semi-infinite soil is evaluated rigorously, b) foundation shape is modeled in detail and c) the method is practical. One of the proposals as a practical method to calculate \( K(0) \) and \( K'(\omega_1) \) is shown here that is based on the wave propagation theory. It is necessary to use soil properties obtained by equivalent linearization for strain dependent properties. The subscripts \( s \) and \( r \) in the following indicate sway and rocking motion, respectively.

The stiffness of sway and rocking of flat foundation on ground surface are obtained as follows.

\[
K_{bs} = \frac{8G r_s}{2 - \nu}, \quad r_s = \sqrt{\frac{A_t}{\pi}}
\]  \hspace{1cm} (7.2.55a)
\[ K_{br} = \frac{8G}{3(1-\nu)} \left( \frac{r}{r_t} \right)^3, \quad r_t = \sqrt{\frac{4}{\pi} I_l} \]  

(7.2.55b)

where \( A_t \): area of base, \( I_l \): second moment of cross-section, \( G \): shear modulus, \( \nu \): Poisson’s ratio, and \( \rho, V_s \): density and shear wave velocity of soil.

The imaginary part of the impedance in Eq. (7.2.53) is obtained as follows\(^{22, 23}\).

\[
K'_{bs} = 2\xi_g K_{bs} \quad (\omega_1 \leq \omega_g) \\
2\xi_g K_{bs} + (\omega_1 - \omega_g) \rho V_s A_t \quad (\omega_1 > \omega_g)
\]  

(7.2.56a)

\[
K'_{br} = 2\xi_g K_{br} \quad (\omega_1 \leq 2\omega_g) \\
2\xi_g K_{br} + (\omega_1 - 2\omega_g) \rho V_L I_l \quad (\omega_1 > 2\omega_g)
\]  

(7.2.56b)

\[
V_L = \frac{3.4V_s}{\pi(1-\nu)}
\]  

(7.2.57)

where \( \xi_g \): hysteretic damping ratio, \( \omega_g \): the predominant circular frequency of a layered soil (0 in uniform soil).

In the case of actual multi-layered soil, the soil parameter of \( G \) in Eq. (7.2.55) is obtained by the modification of Eq. (7.2.58)\(^{24}\). Here the soil is composed of layers with different soil parameters in \( k = 1 \sim n \) (\( k = n \) means engineering bedrock) shown as Fig.7.2.11.

\[
\frac{1}{G} = \sum_{k=1}^{n} \frac{1}{G_k} \left( F_{bs}(z_{k-1}/r_s) - F_{bs}(z_k/r_s) \right),
\]  

(7.2.58a)

where \( z_0 = 0 \), \( F_{bs}(z_{n}/r_s) = 0 \). The parameter \( V_s \) in Eq. (7.2.56) is obtained as \( \sqrt{G/\rho} \) using \( G \) in Eq. (7.2.58) and the mean value of \( \rho \). The parameter \( \xi_g \) in Eq. (7.2.56) is modified as follows.

\[
\xi_g = \sum_{k=1}^{n} \frac{G}{G_k} \xi_g \left( F_{bs}(z_{k-1}/r_s) - F_{bs}(z_k/r_s) \right)
\]  

(7.2.59)

where \( \xi_{gk} \) is the equivalent viscous damping ratio of \( k \)-th layer.

As for embedded foundations, the impedance function is obtained as the sum of impedances of side wall and bottom plane where the equations of (7.2.55) and (7.2.56) are applied using the soil property of soil under the bottom plane. The impedance for side wall is obtained as follows\(^{25, 26}\).

\[
K_{ws} = 2\eta K_{bs}, \quad K_{wr} = (2.6\eta + 5.6\eta^3)K_{br}, \quad \eta = \frac{d}{\sqrt{A_t}}
\]  

(7.2.60)

\[
K'_{ws} = 2\xi_g K_{bs} + \omega_1 (\rho V_s A_1 + \rho V_L A_2)
\]  

(7.2.61)

where \( d \) is the depth of side wall, \( A_1, A_2 \) are the area of parallel and perpendicular side wall against the direction of motion, respectively. To calculate \( K_{bs}, K_{br} \) the average soil property of surface ground i.e. soil supporting side-wall is applied.
7.2. Calculation of Seismic Loads

As for pile foundations\textsuperscript{27, 28)}, the stiffness for sway is obtained on the basis of the beam theory\textsuperscript{29)} on an elastic medium as follows.

\[ K_{ps} = N_p \frac{4E_p I_p \beta^3}{2 - \alpha} \]  

(7.2.62)

where \( E_p, I_p \): the Young’s modulus and the sectional moment of inertia of the pile, \( N_p \): the number of piles, \( \alpha = 1, 0 \): fixed, pin support condition at pile top. The characteristic value of \( \beta \) is obtained as follows.

\[ \beta = \sqrt[4]{\gamma_p k_g 4E_p I_p} \]  

(7.2.63)

The damping of piles as the imaginary part of impedance is obtained as follows\textsuperscript{30)}.

\[ K'_{ps} = \begin{cases} 
1.5 \zeta_g K_{ps} & \text{for } \omega_1 \leq \omega_{g1} \\
1.5 \zeta_g K_{ps} + (\omega_1 - \omega_{g1})0.75 \gamma_p 0.75 \frac{c_g}{k_g} K_{ps} & \text{for } \omega_1 > \omega_{g1} 
\end{cases} \]  

(7.2.64)

The stiffness \( k_g \) and damping \( c_g \) per unit length of pile is obtained as follows.

\[ k_g = \frac{1.3E_s}{1 - \nu^2} \frac{12}{E_p I_p} D_p^4, \quad c_g = 0.5\pi \rho D_p (V_s + V_L) \]  

(7.2.65 a, b)

where \( D_p \): the diameter of a pile, \( E_s \): the Young’s modulus of soil.

The parameter \( \zeta_g \) is modified as follows.

\[ \zeta_g = \sum_{k=1}^{n'} \frac{1}{\beta_k^3} \left\{ F_p(z_{k-1}) - F_p(z_k) \right\} \]  

(7.2.66)

\[ F_p(z) = \exp(-\beta z) \cos(\beta z) \]  

(7.2.67a)

\[ F_p(z_{n'}) \equiv F_p(z_{n'}) \]  

(7.2.67b)

\[ \beta = \frac{0.5\pi}{\bar{z}_{n'}} \bar{z}_{n'} = \bar{z}_{n'-1} + \bar{d}_{n'} \quad (\leq \text{pile length}) \]  

(7.2.67c)

where \( \bar{z}_{n'} \) is determined to be \( 0.5\pi \). The parameters \( k_g, c_g \) in Eq.(7.2.64) are converted from Eq.(7.2.63). The parameter \( \zeta_g \) is modified as follows.

\[ \zeta_g = \sum_{k=1}^{n'} \left( \frac{\beta}{\beta_k} \right)^{0.75} \zeta_{gk} \left\{ F_p(z_{k-1}) - F_p(z_k) \right\} \]  

(7.2.68)

The stiffness for rocking is obtained considering axial deformation of a pile and reaction from surrounding and supporting soil. The damping for rocking in Eq.(7.9) is generally less than the damping for sway.

7.2.2 Acceleration response spectrum

1) Basis of determination of acceleration response spectrum

This section illustrates how to determine the acceleration response spectrum, which is often used in conventional seismic design, that is, an input ground motion to the structural model (an SSI model and a fixed-base model) described in Sec.7.2.1.

The determination of the acceleration response spectrum requires adequate consideration of ground properties affecting the earthquake motions, vibrational properties of a building and other characteristics. Figs.7.2.12 and 7.2.13 depict schematic relationship of these factors. These seismic loads are
determined so that these factors are taken into consideration as realistically as possible\(^{31}\). To do so, available information and database related to site characteristics and a building must be fully utilized, as is shown in Table 7.1.1.

It is necessary to select the best determination method depending upon quality an quantity of available information. It is not desirable to adopt a particular method with disregard to those of the information. It is essential to collect the information of the ground as much as possible, and to carefully select the building model.

<table>
<thead>
<tr>
<th>Seismic Load Effect</th>
<th>Source Characteristic</th>
<th>Ground Motion Wave Amplification</th>
<th>Soil-Structure Interaction</th>
<th>Building Vibrational Characteristic</th>
</tr>
</thead>
</table>

**Fig.7.2.12** Factors affecting seismic load

In these recommendations, the standard procedure to determine the acceleration response spectrum is provided. However, when the seismic load to a building is determined approximately, or the detailed procedure cannot be used, a simplified procedure is also shown. Note that the simplified procedure should be adopted only in the case where there is little information and the accuracy of this procedure is not as good as the detailed one. Furthermore, the acceleration response spectrum described in the previous recommendations (1993) can also be used to determine the seismic load in a simple manner.

Figs.7.2.12 and 7.2.13 illustrate several factors which affect the seismic load in a building. The calculation procedure of the seismic design load (an acceleration response spectrum to the building) is dependent upon how rigorously these factors are treated. Table 7.2.1 shows the possible procedures: recommended, detailed and simplified procedures, and ways of treating the factors. In the current recommendations, the way of treatment is not specified so that a designer can make the best use of his ability and available information.

**Fig.7.2.13** Factors affecting seismic load
### Table 7.2.1 Factors affecting response spectrum

<table>
<thead>
<tr>
<th>Modification Factor</th>
<th>Recommended Procedure</th>
<th>Detailed Procedure</th>
<th>Simplified Procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSI (effective input motion)</td>
<td>$H_{SSI}(\omega)$</td>
<td>approximated equation of Eq.(7.17)</td>
<td>size and shape of foundation and embedment</td>
</tr>
<tr>
<td>Wave amplification</td>
<td>$H_{GS}(\omega)$</td>
<td>1D shear wave propagation analysis considering strain dependency of soil stiffness and damping ratio</td>
<td>layer composition, shear wave velocity, strain-dependency of stiffness and damping of layers above the engineering bedrock</td>
</tr>
</tbody>
</table>

2) Power spectral representation of ground motion

Fourier’s analysis dictates that a time-varying process can be expressed in terms of superposition of various sinusoidal waves with different amplitudes. On the other hand, in the field of engineering, a response spectrum is often used to represent the ground motion properties. The response spectrum originally do not represent the spectral characteristics of ground motions directly, but represents the maximum response of a linear single-degree-of-freedom system with a specified period and damping ratio, and its representation has been adopted for the engineering purpose. However, it has the shortcoming that it cannot be decomposed into several factors affecting the maximum response of an SDOF, as shown in Fig.7.2.12.

On the other hand, power spectra (power spectral density or power spectral density function) does express the energy contribution along the frequency range of a mean squared time-varying process, and the power spectral representation has mathematical advantage. The advantage is that all factors affecting the maximum response can be described in an functional form in a frequency range. Once the earthquake ground motion is described in terms of a power spectra, each factor illustrated in Fig.7.2.12 can easily be incorporated in a simple mathematical form, as is shown by Eq.(7.13). Transfer functions associated with each factor can be evaluated in several ways that are currently available.

As is shown in Fig.7.2.13, the input ground motion to a building can be finally estimated by multiplying a basic power spectrum of the ground motion on the bedrock by a wave amplification function and a soil-structure interaction function. Furthermore, a power spectrum of the input ground motion can be directly used for the dynamic response analysis in a frequency domain.

When focusing on the ultimate limit state of a building, however, nonlinear response of the building must be estimated, and therefore, representation of response spectra of the input ground motion, which have been familiar so far, would be desirable. If so, it is needed to transform the power spectrum into the corresponding response spectrum and the transformed response spectrum can be used in various ways as usual.
3) Transformation of response and power spectra

Fig. 7.2.14 illustrates the composition of the seismic load and interrelationship among the factors. The input ground motion to a building as well as the ground motion on the engineering bedrock are characterized in a response spectra. It requires transformation of response and power spectra. When a time history of a ground motion is available, it is quite easy to evaluate the response and power spectra. Evaluation of the time histories is given in Sec. 7.3. The maximum amplitude of a ground motion can be stochastically estimated by assuming a main part of the ground motion to be a stationary process with a finite duration time. Using this estimation theory, the maximum response of a specified SDOF system subjected to a ground motion where the power spectrum is given can be predicted with relatively allowable accuracy. This theory is called “transformation of power and response spectra” and several methods have been previously proposed. A method for the spectral transformation based on the first excursion theory of a stationary random process is briefly written below. For more details, see Reference 32. The transformation from a power to response spectra is called “a forward transformation” hereafter and the reverse is called “a backward transformation”.

**Fig. 7.2.14**  Composition of seismic load and factors affecting response spectra to each model

**a) First excursion theory**

An elapsed time $T_d$ that a random process $X(t)$ first exceeds a specified threshold becomes a random variable and to estimate the probability density function of the elapsed time is called “a first excursion theory”. From the different point of view on this problem, it is equivalent to the estimation of the probability density function of the maximum value of the random process within a specified time.
7.2. Calculation of Seismic Loads

The appropriate solution of this problem was derived by Vanmarcke\textsuperscript{33, 34), and is given in Eq.(7.2.69).

\[ X_{\text{max}}(T_d) = \gamma(T_d, p)\sigma_X \]  

(7.2.69)

where \( X(t) \): a stationary Gaussian random process with zero mean with a duration time \( T_d \), \( X_{\text{max}}(T_d) \): a maxima of the process within \( T_d \), \( \sigma_X \): a standard deviation of \( X(t) \) and \( \gamma(T_d, p) \): a peak factor with a non-exceedance probability \( p \).

The peak factor \( \gamma(T_d, p) \) is a function of the duration time and the non-exceedance probability and its expected value is around 3.0 for ordinary ground motions. The peak factor is highly related to the frequency content of the process. Fig.7.2.15 depicts the excursions of the process with both-side barriers. Although the several peak factors have been proposed so far, the following solution proposed by Vanmarcke is taken. The peak factor of a stationary Gaussian random process with a duration time \( T_d \) is given in the following approximate equation,

\[ \gamma(T_d, p) = 2 \ln \left[ 2n \left( 1 - \exp \left( -\delta^2 \sqrt{\frac{\ln 2}{\ln n}} \right) \right) \right]^{1/2} \]  

(7.2.70)

\[ n = \frac{\Omega_x T_d}{2\pi} \left( -\frac{1}{\ln p} \right) \]  

(7.2.71)

where \( p \) is a specified non-exceedance probability, \( \Omega_x \) is a characteristic frequency and \( \delta_x \) is a bandwidth of the random process \( X(t) \). \( \delta_x \) is an index to control the scatter of the power spectrum of the process.

b) Spectrum transformation

Using the first excursion theory, the spectrum transformation can be performed. From a given power spectrum of a ground motion, the response spectrum can be estimated based on the maximum response of an SDOF system subjected to the ground motion.

It is slightly cumbersome to transform a given response spectrum into the power spectrum. Since the forward transformation can easily be performed, the backward transformation can be done through several iteration of the forward transformation. Fig.7.2.16 shows some results from the forward transformation (from the given power spectrum to the response spectrum) along with the results from Monte Carlo simulations in which one hundred time histories with given power spectrum are generated and all response spectra are calculated.

The maximum response of a linear oscillator with a natural circular frequency \( \Omega_s \) and a damping ratio \( \zeta_s \) subjected to a ground motion, which is the definition of the response spectrum, can be derived based on the first excursion theory.
The power spectrum of the input to a system $G_I(\omega)$ are related to the power spectrum of the system response with the transfer function of the system $H(\omega)$ as follows.

$$G_O(\omega) = |H(\omega)|^2 G_I(\omega)$$  \hspace{1cm} (7.2.72)

where $H(\omega)$ is the acceleration transfer function to the acceleration input and is given as

$$|H(\omega)|^2 = \frac{\omega_s^4 + 4\zeta_s^2 \omega_s^2 \omega^2}{(\omega_s^2 - \omega^2)^2 + 4\zeta_s^2 \omega_s^2 \omega^2}$$  \hspace{1cm} (7.2.73)

By using Eq.(7.2.72), the standard deviation of the response of the oscillator can be derived as

$$\sigma^2(\omega, \zeta) = \int_0^{\infty} |H(\omega)|^2 G_I(\omega) d\omega$$  \hspace{1cm} (7.2.74)

In the subsequent discussion, a procedure to transform the given response spectrum into the corresponding power spectrum will be illustrated. This backward transformation can be performed based on the iterative technique of the forward transformation. From the appropriately selected initial power spectrum, the iteration of the forward transformation will be started and repeated until the resulting response spectrum becomes the one specified as a target response spectrum within an acceptable error.

Fig.7.2.16 shows the mean response spectra ($p = 50\%$) of one hundred stationary processes with random phases and with the Kanai-Tajimi power spectra along with the mean response spectra ($p = 50\%$) from the forward transformation directly from the given power spectrum. Two duration time $T_d$, 5s and 40s, are used and the damping ratios of the oscillator are changed from 0.01 to 0.5. From these figures, the spectrum transformation based on the first excursion theory of stationary processes works quite well. More careful observation indicates that for smaller damping ratios, shorter duration time, and longer natural period of the oscillator, the accuracy of the transformation decreases. Such accuracy of the transformation is closely related to the applicability of the peak factor as mentioned in the next section c).
7.2. Calculation of Seismic Loads

c) Applicable range of Vanmarcke’s peak factor\(^{32}\)

There is an applicable range of the Vanmarcke’s peak factor. For example, the shape factor of the random process for an extremely small damping ratio approaches zero and the resulting peak factor becomes zero. Furthermore, for longer natural period of an oscillator, the characteristic circular frequency becomes smaller and the peak factor becomes zero. When the duration time is very short, the peak factor becomes unstable.

Fig. 7.2.17 indicates the applicable limit (boundary of \(\gamma = 0\)) of the Vanmarcke’s peak factor in the \(n-\delta_X\) plane. It can be seen that the applicable range becomes wider when both \(n\) and \(\delta_X\) are larger.

4) Response spectrum of the previous recommendations (1993)

In the previous recommendations (1993), the response spectrum was defined at the surface of the ground as follows.

\[
S_a(T, \zeta) = \begin{cases} 
(1 + \frac{f_A - 1}{d} \frac{T}{T_c}) F_h G_A R_A A_0 & 0 \leq T \leq dT_c \\
F_h f_A G_A R_A A_0 & dT_c \leq T \leq T_c \\
\frac{2\pi F_h f_V G_V R_V V_0}{T} & T_c \leq T
\end{cases}
\]  

(7.2.75)

where

- \(f_A\): ratio of \(S_a(T, 0.05)\) to \(G_A R_A A_0\) for \(dT_c \leq T \leq T_c\),
- \(f_V\): ratio of \(S_a(T, 0.05) = S_a(T, 0.05)T/2\pi\) to \(G_V R_V V_0\) for \(T_c \leq T\),
- \(d\): ratio of the lower bound period to upper bound period of the range where \(S_a(T, \zeta)\) is constant,
- \(T_c\): upper bound period of the range where \(S_a(T, \zeta)\) is constant, and given by Eq.(7.2.76),
- \(F_h\): damping modification factor,
- \(A_0\): basic peak acceleration of earthquake ground motion at the reference soil,
- \(V_0\): basic peak velocity of earthquake ground motion at the reference soil,
- \(R_A\): return period conversion factor for the peak acceleration of earthquake ground motion,
- \(R_V\): return period conversion factor for the peak velocity of earthquake ground motion,
- \(G_A\): soil type modification factor for the peak acceleration of earthquake ground motion,
- \(G_V\): soil type modification factor for the peak velocity of earthquake ground motion.

\[
T_c = \frac{2\pi F_h f_V G_V R_V V_0}{f_A G_A R_A A_0}
\]

(7.2.76)

The response spectrum of Eq.(7.2.75) specifies the appropriate acceleration response based on the spectral property of the ground motion and on the natural period and damping ratio of the structure, and includes but does not explicitly represent factors such as earthquake sources, propagation path and wave amplification in a ground soil. The acceleration spectrum consists of an acceleration constant range \((dT_c < T < T_c)\) and a velocity constant range \((T > T_c)\), and the level of the former range
is \( f_A A_0 \) and that of the latter range is \( f_V V_0 \) for basic peak acceleration and velocity, \( A_0 \) and \( V_0 \), respectively. As for other modifications, soil type modification factors \( G_A \), \( G_V \), a damping modification factor \( F_h \) and return period conversion factors \( R_A \) and \( R_V \) to the return period except for 100 years. The previous version recommended the following values.

\[ f_A = 2.5 \ , \ f_V = 2.0 \ , \ d = 0.5 \]  

(7.2.77)

\( G_A \) and \( G_V \) are given in Table 7.2.2.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>( G_A )</th>
<th>( G_V )</th>
<th>( T_c ) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I: Hard (Standard)</td>
<td>1.0</td>
<td>1.0</td>
<td>0.33</td>
</tr>
<tr>
<td>Type II: Soft Diluvial or Dense Alluvial</td>
<td>1.2</td>
<td>2.0</td>
<td>0.56</td>
</tr>
<tr>
<td>Type III: Soft</td>
<td>1.2</td>
<td>3.0</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Furthermore, \( F_h \) is the same as \( \mu_d \) in Eq.(7.2.20). The basic peak acceleration can be \( a_0 \) as specified in Sec.7.2.2(4), and the return period conversion factor can be \( k_{rE} \) in Sec.7.2.2(5).

(1) Acceleration power spectrum at the engineering bedrock

A basic peak ground acceleration at the engineering bedrock \( a_0 \) is given in the form of map shown in section Sec.7.2.2(4). In addition, the frequency characteristic of the ground motions is needed. Here, a spectral shape \( S_0(T, 0.05) \) will be specified although the frequency characteristics of the ground motions on the engineering bedrock with the shear wave velocity of 400m/s has been reported so far. According to the assumptions of constant acceleration and constant velocity period ranges in the response spectra, Eq.(7.14) in the main text is used and its spectral shape is roughly drawn in Fig.7.2.18.

In case of 5 percent damping ratio, the response ratio \( k_{R0} \) is between 2 and 3, and the corner period between the constant acceleration range and the constant velocity range is specified as \( T_c' / T_c = 1/5 \sim 1/2 \). When the spectrum transformation is performed, the normalized response spectrum (depicted by Fig.7.2.18) can be defined as the response spectrum on the engineering bedrock since the response at \( T = 0 \) is equal to the peak acceleration of the ground motion.

From the basic acceleration response spectrum \( a_0 S_0(T, 0.05) \) as specified above is now transformed into the corresponding power spectrum \( G_{a0}(\omega) \) which can be computed by the backward transformation mentioned before. During the transformation, an effective duration time \( T_d \) is known to be earthquake magnitude-dependent. From the past experience, however, \( T_d \) is set to 20s. The resulting power spectrum \( G_{a0}(\omega) \) will be multiplied by several functions in the frequency domain, and finally the power spectrum of the input ground motion to the structural system model can be estimated.
(2) Soil amplification function

The soil amplification function $H_{GS}(\omega)$ to express the amplification effect of seismic motion through soil deposits is a transfer function, which is a frequency-dependent ratio of the free field motion at the ground level or the base of foundation to the exposed surface motion at engineering bedrock. The soil amplification function is evaluated by the transfer function that is the theoretical solution of a one-dimensional shear wave propagation analysis taking into account soil nonlinearity by the strain-dependent soil properties of shear modulus and damping ratio. Note that, since the transfer function is a complex function, the square of its absolute value is the definition of the soil amplification function, and that the engineering bedrock should be set to the level deeper than the upper level of bearing stratum with sufficient stiffness and thickness and might be expected practically that its shear wave velocity is equal to or more than 400m/s.

Alternatively one can substitute the simplified formula for the soil nonlinearity based transfer function mentioned above. The simplified formula, as shown below, takes into account not the soil nonlinearity but hysteretic damping and is an algebraic expression corresponding to the solution to the soil model with two layers above and below the internal surface of soil deposits and an engineering bedrock. In one-dimensional shear wave propagation analysis, the seismic motion vertically propagating in the horizontally layered soil deposits is expressed as a weighted sum of upward and downward waves with harmonic oscillations at a point. Upward waves are equal to downward waves in amplitude at a free field or an exposed surface. Then, for the two-layer soil model, a simplified formula of the transfer function is derived and given by\textsuperscript{e.g.19, 36}:

$$H_{GS}(\omega) = \frac{1}{\cos A + i \alpha_G \sin A} \quad (7.2.78)$$

$$A = \frac{\omega T_G}{4 \sqrt{1 + 2 i \zeta_G}} \quad (7.16)$$

where $\alpha_G$ represents the impedance ratio from the engineering bedrock to surface soil, $T_G$ and $\zeta_G$ represent respectively the predominant period in seconds and the damping ratio of soil above the engineering bedrock, and $i = \sqrt{-1}$.

It is convenient and useful in a way to adopt the simplified formula in order to obtain the soil amplification function since one can just decide a soil type of site without much information. There is the soil type classification and the reference value of the parameters for each soil type shown in Table 7.2.3. It is to be noted that, in this table, the predominant period is set with reference to that written in the previous Recommendations for Loads on Buildings (1993), the damping ratio is set with reference to the exploration results\textsuperscript{37} of soil deposits with the shear wave velocity in the range of 100~400 m/s, and the impedance ratio is set with reference to the boring exploration data\textsuperscript{38, 39} under the assumption that the damping ratio of the upper layer is equal to that of lower layer. Fig.7.2.19 shows the absolute value of the transfer function corresponding to each soil type. Note that in this figure the abscissa is not circular frequency $\omega$ (rad/s) but period $T(s) \equiv 2\pi/\omega$. However, it is desirable that if the site exploration data are available for parameter setting, one should use readily the site-specific value of the parameters instead of the reference value shown in the table.

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>$T_G(s)$</th>
<th>$\zeta_G$</th>
<th>$\alpha_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I: Hard (Standard)</td>
<td>0.22</td>
<td>0.05</td>
<td>0.50</td>
</tr>
<tr>
<td>Type II: Loose Diluvial or Dense Alluvial</td>
<td>0.37</td>
<td>0.05</td>
<td>0.35</td>
</tr>
<tr>
<td>Type III: Soft</td>
<td>0.56</td>
<td>0.05</td>
<td>0.20</td>
</tr>
</tbody>
</table>
(3) Adjustment function of soil-structure interaction

The behavior of embedded foundation during earthquake is different in general against the behavior of free field ground without foundation. The behavior of a foundation under the assumption to be massless is one of the effects of dynamic soil-structure interaction, which is called as foundation input motion. For the evaluation of seismic load, the effect can be considered as an adjustment factor for acceleration response spectrum.

The input motion is different from free field motion because the free field motion is not constant along the depth and the displacement of soil is restricted by the foundation assumed to be rigid. The concept is explained using Fig.7.2.20.

When free field motion is $U_g(z)$, the resultant force $F_c$ restricted by the foundation becomes

$$F_c = K_{bs} U_g(d) + \int_0^d k_{ws}(z) U_g(z) \, dz$$

where $K_{bs}, k_{ws}(z)$ : springs at base and side wall including wave dissipation effect.

The foundation input motion $U_{fh}$ is obtained by releasing the force as follows.

$$U_{fh} = \frac{F_c}{K_f}$$  \hspace{1cm} (7.2.79)

$$K_f = K_{bs} + \int_0^d k_{ws}(z) \, dz$$

Assuming $k_{ws}$ to be constant, the input motion is expressed as follows.

$$U_{fh} = \frac{K_{bs} U_g(d) + K_{ws} \bar{U}_{gw}}{K_{bs} + K_{ws}}$$  \hspace{1cm} (7.2.80)

$$K_{ws} = dk_{ws}, \quad \bar{U}_{gw} = \frac{1}{d} \int_0^d U_g(z) \, dz$$

From Eq.(7.2.80) the input motion is regarded as the motion averaging $U_g(z)$ with weighting soil spring. Since $U_g(z)$ is less than $U_{GL}$, $U_{fh}$ becomes less than $U_{GL}$. The input motion in long period is equivalent with free field motion as the wave length is long.
A soil structure interaction effect was investigated using observed records during the 1995 Hyogo-ken Nanbu Earthquake\textsuperscript{40}. The relationships of maximum acceleration and velocity between records at free field and at foundation are compiled in Fig.7.2.21. The white circles are pure recorded results and the free field motion in black circles is inversely evaluated from the analyses. The factor $\eta$ in A and B buildings are about 0.3 and 0.5, respectively. The average ratios of $U_{fh}$ to $U_{GL}$ from the least square method was obtained to be 0.7 for acceleration and 0.9 for velocity. These results show that compared to $U_{GL}$, $U_{fh}$ in long period range is a little smaller and the difference in short period becomes smaller.

![Fig.7.2.21 Interation effects of observed records during Hyogoken Nanbu earthquake\textsuperscript{40}]()
From the observed results in Fig. 7.2.21 and analytical results in Fig. 7.2.22, the adjustment factor $H_{SSI}(\omega)$ is defined as follows.

$$\left| H_{SSI}(\omega) \right|^2 = \left| \frac{U_{fh}}{U_{GL}} \right|^2 = \left\{ \begin{array}{ll} 1 & : \delta_d \leq 1 \\ 1 + 2 \eta \delta_d^2 & : \delta_d > 1 \end{array} \right.$$  \hspace{1cm} (7.17)

where $\eta = d/l$, $\delta_d = \omega/\omega_d$, $\omega_d = (\pi V_s)/(2d)$, $l$: equivalent foundation width ($= \sqrt{A_f}$), $d$: embedment depth of foundation, $V_s$: shear wave velocity of the soil adjacent to side wall.

Examples of $H_{SSI}(\omega)$ are shown in Fig. 7.2.24. The rotational input motion should be considered in case it is significant.

**Fig. 7.2.24** Adjustment function of embedded foundation

(4) Basic peak acceleration

1) Seismic hazard and the analytical method

The basic parameter to decide the seismic load of a building is the strength of strong ground motions at the construction site during the lifetime of the building. It is more appropriate to estimate the maximum ground motion stochastically than to determine it deterministically because the strength of strong ground motions in the future depends on various factors with large randomness such as the seismic source characteristic, the wave path characteristic of seismic waves and the site soil characteristics.

The seismic hazard (seismic risk) represents stochastically the strength of seismic motion on the ground or the seismic bedrock that is caused by future earthquakes and the peak ground acceleration or the peak ground velocity with a return period of 100 years, etc. is used as a measure of the seismic hazard.

One of initial researches on the seismic hazard is the “Kawasumi hazard map” (1951) and some analytical methods are developed using different models of seismic sources and data after his research^{42}.
7.2. Calculation of Seismic Loads

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**Fig. 7.2.25** Seismic hazard map: acceleration $a_{500}$ (m/s²)

(Maximum horizontal acceleration at the surface of the engineering bedrock with a return period of 500 years.)

Note) Thick contour lines are drawn every 2m/s² and thin curve every 1m/s². “+” indicates that the value in that area is greater than that of the adjacent area.

---

a) **Seismic hazard analysis using the earthquake catalogue**

The past seismic data (occurrence date and time, location and magnitude) in Japan have been collected in the earthquake catalogue (database) for more than 1,000 years. The recent data within 100 years are based on the records of seismographs, but the other data were estimated through earthquake damages in various documents.

The strength of earthquake motion on a certain point can be estimated using seismic data from the earthquake catalogue and appropriate attenuation relationship (distance attenuation curve). It is preferable to consider the randomness of attenuation relationship because it is an empirical relationship based on numerous observation records and is composed of only a few factors such as the magnitude and the epicentral distance or the hypocentral distance. Though the seismic hazard can be estimated from the distribution function applied to the sample distribution of earthquake motion values, the applied distribution function is not sensitive to large values of the sample distribution.

A more reasonable and improved method is to use the extreme value distribution which are com-
posed of the annual maximum values. The extreme value distribution commonly used are the Gumbel distribution, the Fréchet distribution and the Weibull distribution\(^{43}\) which satisfy the stability condition that the shape of distribution does not change with the size of the group or the distribution proposed by Kanda\(^{44}\) which has the minimum and maximum limits.

To use the extreme value distribution, it is necessary to extend the period of the earthquake catalogue long enough to reflect the influence of a big earthquake in short distance because the recurrence period of a large earthquake is several thousand years or more. However, the setting of long period of the earthquake catalogue is usually difficult because the reliability of the old earthquake data is low.

In the previous recommendations (1993), the seismic hazard is calculated by the method of using the extreme value distribution for the reasons of ease and comprehensiveness of the method. The maximum velocities for the return period of 100 years on the seismic bedrock at points divided by 0.5 degrees of latitude and longitude are estimated by the Fréchet distribution of annual maximum velocities using the Kanai attenuation relationship and the Usami earthquake catalogue and then they are converted into the peak ground acceleration.

These results are significantly different whether a large earthquake is contained or not in the earthquake catalogue. This problem can be avoided by using the seismotectonic zone which has the same characteristic of earthquake occurrence. This analytical method (b value model) is based on the research by Cornell\(^{45}\) and uses the magnitude distribution which is basically the exponential distribution based on the Gutenberg and Richter law and the occurrence model of large earthquakes which is generally assumed as a Poisson process. The seismic hazard is estimated by the probability calculation using data of the maximum magnitude, b value, and average earthquake occurrence rate and an appropriate attenuation relationship. This method can easily take into account the randomness of the distance attenuation relationship.

In the current recommendations, the b value model is used for earthquakes on land where the earthquake activity level is not large (background earthquake activity region).

b) Seismic hazard analysis using earthquake data from the plate boundary and active fault

It is well known that large earthquakes occur repeatedly in the plate boundary or active fault regions. The characteristic of occurrence of earthquakes in such regions shows that large earthquakes occur repeatedly at almost the same recurrence period. Such characteristic suggests that the peculiar earthquake model with the same scale earthquakes is more appropriate than the b value model with different scale earthquakes. Recently, the seismic hazard analysis using the peculiar earthquake model is improved as the data on the position of the active fault, shape, the magnitude of the earthquake, and the recurrence interval are accumulated.

The problem with this method is that the reliability of the data from the plate boundary regions or the active fault regions in Japan are not the same. Moreover, the reliability of the average displacement rate is low because the recurrence interval of large earthquakes is several thousand years or more.

In the current recommendations, the peculiar earthquake model is used for earthquakes in the plate boundary and active fault regions.

c) Seismic hazard analysis in the current recommendations

In the current recommendations, the seismic hazard is analyzed by two models; the b value model and the peculiar earthquake model, according to seismic regions and the results are integrated. The details of the seismic occurrence data and the analysis method are shown in the appendix and an outline is shown here. This analysis method is basically based on the research by Annaka\(^{46}\).
7.2. Calculation of Seismic Loads

The peculiar earthquake model is applied in 28 regions based on historical earthquakes and 117 regions based on the plate boundary and active fault regions. The maximum magnitude $M_u$ is not a fixed value but is assumed to be equally distributed between $M_u - 0.2$ and $M_u + 0.2$.

The $b$ value model is applied in 29 districts in the upper side on the Pacific Ocean plate, 7 districts in the lower side on the Pacific Ocean plate, 35 districts in upper side on the Philippine Sea plate and 25 districts in intraplate.

The distance attenuation relationship proposed by Annaka and Yashiro\(^{47}\) is used which shows an average maximum acceleration on the seismic bedrock in Japan. The relationship using the unit m/s\(^2\) is as follows.

\[
\log A = 0.606M + 0.000459h_c - 2.136 \log d - 0.270 \quad (7.2.81a)
\]

\[
d = R + 0.334 \exp [0.653M] \quad (7.2.81b)
\]

where $A$ is the maximum acceleration on the seismic bedrock (m/s\(^2\)), $M$ is the Japan Meteorological Agency magnitude, $R$ is the shortest distance from the source (km), and $h_c$ is the mean depth of the source (if it is beyond 100 km, then $h_c$ is 100 km). Moreover, the logarithmic standard deviation which represents the randomness of the attenuation relationship is assumed 0.5 in the natural logarithmic scale.

The interval of mesh points where earthquakes occur is assumed to be 5 km. The extreme value distributions of annual maximum acceleration are calculated at points of every 15 minutes in longitude and latitude. The maximum acceleration with 100 year return period (probability is 0.99) and the maximum acceleration with 500 year return period (probability is 0.998) are obtained from the distribution. These hazard maps are shown in Fig.7.1 and Fig.7.2.25.

d) Estimation of the extreme value distribution and randomness of seismic hazard

Although values with return periods of 100 and 500 years are obtained from the hazard maps, it is necessary to obtain the extreme value distribution for estimating other return period value or for use in reliability analysis.

The extreme value distribution can be obtained by using two hazard maps. Two values of return periods of 100 and 500 years in maps will determine two parameters of the Gumbel distribution, and two parameters of the Fréchet distribution when the lower bound value assumed is zero, and two parameters of the Weibull distribution when the upper bound value is assumed to be large.

It is not easy to represent the standard deviation of $N$ year return period value because the value is not the statistical moment but the percentage of the probability distribution\(^{48}\).

When it is necessary to obtain the randomness of the seismic hazard, the randomness of the R year maximum value is often used. For instance, the value of 100 year return period is equal to the mean of the 56 year maximum value in the Gumbel distribution, and the value between the mean of the 47 year maximum value and the mean of the 51 year maximum value in the Fréchet distribution when the shape coefficient $k$ is between 5 and 10. Therefore, the randomness of the value of 100 year return period can be used to approximate the randomness of the 50 year maximum value.

2) Coefficient of variation of 50-year maximum acceleration

The coefficient of variation of 50-year maximum acceleration shown in Fig.7.2.26 is obtained by fitting the hazard curve at a point on the extreme value distribution. As there are cases that the coefficient of variation cannot be obtained for the Fréchet distribution, the extreme value distribution
with upper and lower bounds is used for the fitting. The range of the fitting is from 0.1 to 30m/s\(^2\) and the upper bound is set to 35m/s\(^2\).

![Fig.7.2.26 Map of coefficient of variation of 50 year maximum acceleration, using the extreme distribution with upper and lower bounds](image)

(5) Return period conversion factor

1) Conversion factor of return period \(k_{rE}\) can be obtained using the basic peak acceleration \(a_0\) and the peak horizontal acceleration for 500-year return period \(a_{500}\) if the type of probabilistic distribution is assumed. When a random variable \(X\) follows the Fréchet distribution, CDF \(F_X(x)\) is given by:

\[
F_X(x) = \exp\left(-\left(\frac{v}{x}\right)^k\right)
\]  

(7.2.82)

where \(v\) and \(k\) are parameters of the distribution. Taking the natural logarithm of both sides twice and rearranging gives

\[
\ln\left(-\ln F_X(x)\right) = -k (\ln x - \ln v)
\]

(7.2.83)

When the probability of exceedance \(1 - F_X(x) \ll 1\) then \(1 - F_X(x) \equiv -\ln F_X(x)\), the equation above can be approximated by

\[
\ln\left(1 - F_X(x)\right) = -k (\ln x - \ln v)
\]

(7.2.84)
This equation means that the probability of exceedance $1 - F_X(x)$ can be approximated by a line in double-logarithmic scale. As $1 - F_X(x) = 1/r$ for return period $r$, the slope of the line $k$ is given by following equation using the peak accelerations for 100-year return period $a_0$ and 500-year return period $a_{500}$:

$$k = \frac{\ln 500 - \ln 100}{\ln a_{500} - \ln a_0} \approx \frac{1.61}{\ln a_{500} - \ln a_0} \quad (7.2.85)$$

Let us define the peak acceleration for $r$-year return period $a_r$, then the conversion factor of return period $k_{rE}$ is $a_r/a_0$. Replacing 500 with $r$ and $a_{500}$ with $a_r$ in the equation above and rearranging gives the following equation, which is Eq.(2.4) in Chapter 2.

$$k_{rE} = (r/100)^{1/k} \quad (7.2.86)$$

As the conversion factor of return period $k_{rE}$ described above is obtained by linear approximation of the seismic hazard curve between values corresponding to 100 and 500 year return periods, the farther the return period is from the range, the larger the error becomes. If we obtain the conversion factor of return period directly from a seismic hazard curve, such error will decrease. The comparisons of a seismic hazard curve and a linear approximation in Tokyo and Osaka are shown in Fig.7.2.27. In the previous recommendations (1993) $1/k$ of 0.54 is adopted for whole country for the value $r$ between 20 years and 500 years.

![Comparison between seismic hazard curve and linear approximation](image)

Fig.7.2.27  Comparison between seismic hazard curve and linear approximation

2) Failure probability of structure

Seismic load effect $S$ and resistance of structure $R$ are both random variables and can be modeled by probabilistic distribution. The probability of failure of a structure can be evaluated by these probabilistic distributions, but the calculation is usually complicated. Cornell\(^{50}\) showed the probability of failure of a structure $P_f$ is evaluated by following explicit equation when the probabilistic distribution of resistance of a structure is lognormal and a seismic hazard curve is approximated by a line on double logarithmic scale.

$$P_f \approx H(R)e^{\frac{1}{2}(\log R)^2} \quad (7.2.87)$$
where $H(x) = 1 - F_X(x)$ is the seismic hazard curve, $\hat{R}$ is a median of structural resistance, and $\delta_R$ is the lognormal standard deviation of structural resistance. To make failure probability of a structure less than $P_0$, $\hat{R}$ should be as follows:

$$\hat{R} \geq H^{-1}(P_0)e^{\frac{1}{2}k\delta_R^2} = S_{p0}e^{\frac{1}{2}k\delta_R^2}$$  \hspace{1cm} (7.2.88)$$

where $S_{p0}$ is the load corresponding to the failure probability $P_0$ in the seismic hazard curve.

The value of $e^{\frac{1}{2}k\delta_R^2}$ is considered to be a correction factor for peak acceleration to decide design resistance of a structure. The values of $e^{\frac{1}{2}k\delta_R^2}$ are shown in Table 7.2.4 for $k$ and $\delta_R$. The steeper the slope of a seismic hazard curve and the larger the variation of structural resistance becomes, the larger is the correction factor.

| Table 7.2.4 Values of correction factor $e^{\frac{1}{2}k\delta_R^2}$ |
|-----------------|----------|----------|----------|----------|
| $k$             | 0.0      | 0.2      | 0.4      | 0.6      |
| 1.0             | 1.00     | 1.02     | 1.08     | 1.20     |
| 2.0             | 1.00     | 1.04     | 1.17     | 1.43     |
| 3.0             | 1.00     | 1.06     | 1.27     | 1.72     |

(6) Comparison with previous recommendations (1993)

1) Basic peak acceleration in previous recommendations (1993)

The basic peak acceleration in previous recommendations (1993) was obtained by fitting an extreme value distribution to the distribution of annual maximum values given by earthquake catalogues and an attenuation equation. Kanai’s attenuation equation for peak velocity was adopted and the depth of all earthquakes was fixed at 30 km. For the evaluation of local seismic hazard, earthquake data of past 400 years were used and Fréchet distribution was fitted to a Hazen plot of annual maximum velocity. Then the values were multiplied by 1.51, which is the whole country mean ratio of the values using 100 year to 400 year statistic sampling period, to get $V_{k100}$ for 100 year return period. However, the values of 100 year statistic sampling period were adopted for Hokkaido and south of Tanegashima. $V_{k100}$ is the peak velocity on bedrock for 100-year return period and the basic peak velocity $V_0 = 2V_{k100}$ on the standard ground, corresponding to the Type I ground (shear wave velocity of about 400 - 1000m/s). The basic peak acceleration was defined as $A_0 = 15V_0$.

2) Comparison of attenuation equations

An attenuation equation of peak acceleration by Annaka, et al. is adopted in the current recommendations. Annaka, et al. used bedrock motions estimated by the one-dimensional wave propagation theory for observation points whose surface ground shear wave velocity is lower than 300m/s, while observed motions are directly used for other observation points. Therefore, evaluated values correspond to surface ground shear wave velocity higher than 300m/s which is a little lower than that of the lower bound of the standard ground in the previous recommendations (1993). A comparison of attenuation equations is shown in Fig.7.2.28. The peak acceleration values obtained using the method in the previous recommendations, i.e. the values by Kanai’s attenuation equation are multiplied by 30 to be converted to the values of the Type I ground, are compared with the peak acceleration values by the Annaka, et al. attenuation equation. The values obtained by dividing the PGA of the Si and
7.2. Calculation of Seismic Loads

Midorikawa attenuation equation\textsuperscript{52}) (Inter-plate) by the mean ground amplification factor of 1.4 are also shown for comparison. The earthquake magnitude is 7.0 and the focal depth is 30km. The values from the previous recommendations are 1.3 to 1.5 times larger than those by the Annaka, et al. attenuation equation, indicating the previous recommendations gave greater acceleration than the median value of the attenuation equation.

![Comparison of attenuation equations](image)

Fig.7.2.28 Comparison of attenuation equations (Magnitude: 7.0, Depth: 30km)

3) Comparison of peak accelerations in seven cities

A comparison of the peak accelerations on the engineering bedrock in seven cities between the current recommendations and the previous recommendations (1993) is shown in Table 7.2.5. The values of 100 year return period were evaluated using the values in Table 7.2.1 in the previous recommendations. The values of 500 year return period were obtained using the return period conversion factor. The values of the current recommendations are obtained by linear interpolation of the hazard curve at a calculated point closest to each city.

In the limit capacity design of Building Standard Law, seismic loads are evaluated based on an acceleration response spectrum on the engineering bedrock. The peak acceleration values on the engineering bedrock correspond to 0.64m/s\(^2\) for a rare earthquake and 3.2m/s\(^2\) for a very rare earthquake after multiplying by the seismic zoning factor \(Z\). As the return period of seismic loads is not specified in the Building Standard Law, it is not possible to directly compare the values with those of
the current recommendations, but the peak acceleration used in the limit capacity design of Building Standard Law are shown in Table 7.2.5 for comparison.

For the 100 year return period in Table 7.2.5, the values for Sapporo, Sendai, Niigata, and Nagoya are larger than those of the previous recommendations and the value for Tokyo is smaller than that of the previous recommendations. The difference is small in Osaka and Fukuoka. For the 500 year return period, the values for Sapporo, Sendai, and Niigata are larger than those of the previous recommendations and the values are smaller in other cities. Especially, the value of 500 year return period in Tokyo is considerably smaller than that of the previous recommendations.

The main reasons for the difference are considered to be the difference in the seismic hazard evaluation method between the current and previous recommendations (1993) and the sensitivity of statistic values for long return periods such as 100 years (or 500 years) to evaluation method. In other words, a Cornell style method with a probabilistic model of seismic sources is used in the current recommendations, while the fitting of extreme value distribution to annual maximum values using earthquake catalogues is used in the previous recommendations, so the difference in the method is considered to be one of the reasons for the difference in the results. Moreover, a seismic hazard curve is evaluated at each point in the current recommendations, while an average return period conversion factor regardless of location is adopted in the previous recommendations.

<table>
<thead>
<tr>
<th>City</th>
<th>Acceleration for 100 year return period</th>
<th>Acceleration for 500 year return period</th>
<th>Building Standard Law (Limit capacity design)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sapporo</td>
<td>0.95</td>
<td>0.55</td>
<td>2.02</td>
</tr>
<tr>
<td>Sendai</td>
<td>1.76</td>
<td>1.12</td>
<td>3.49</td>
</tr>
<tr>
<td>Niigata</td>
<td>1.52</td>
<td>1.12</td>
<td>3.50</td>
</tr>
<tr>
<td>Tokyo</td>
<td>2.10</td>
<td>2.29</td>
<td>3.61</td>
</tr>
<tr>
<td>Nagoya</td>
<td>2.02</td>
<td>1.77</td>
<td>3.91</td>
</tr>
<tr>
<td>Osaka</td>
<td>1.89</td>
<td>1.92</td>
<td>3.73</td>
</tr>
<tr>
<td>Fukuoka</td>
<td>0.60</td>
<td>0.68</td>
<td>1.38</td>
</tr>
</tbody>
</table>

7.2.3 Reduction factor related to ductility and response deformation

The equivalent static force using the response spectrum method is described in this section, and this method is applied to the building in case most parts are expected to have similar amount of inelasticity as described in the comments in Sec.7.1.1.

1) Reduction factor related to ductility

The seismic shear of the i-th story of a building is estimated using reduction factor $k_{DI}$ related to the ductility of a building which is described in Eq.(7.1) using the procedure with eigenvalue analysis, and in Eq.(7.5) without eigenvalue analysis. However, for the evaluation in the elastic range of a building, this reduction factor is not considered.

The structural characteristic factor $D_s$ of the current Building Standard Law, the R factor of the regulation of the U.S. etc. are equivalent to this reduction factor $k_{DI}$. Equal energy rule or equal displacement rule can be applied.
Moreover, in case that $k_{D_i}$ is considered in relation with an input earthquake motion, time-history of the near-field earthquakes recorded during the 1994 Northridge Earthquake, the 1995 Hyogo-ken-Nanbu Earthquake etc. exhibit pulse-like wave form with very high amplitude. The efficiency of hysteretic damping becomes less than in the case of far-field earthquake, because the inelastic response due to these time-histories tend to increase in one direction. When there is a fault in the near-field region where high probability of activity is expected in the near future, it is recommended that the seismic shear force for design should be higher than that predicted by the method in this recommendation.

1) Reduction factor based on equal energy rule or equal displacement rule

The equal energy rule or the equal displacement rule \(^{54}\) based on the maximum response relationship between elastic and elastoplastic systems having the same initial periods may be used. The strength reduction factor $R_\mu$ and the maximum response displacement $d_p$ is expressed as follows.

For short-period systems ($T < 0.5s$), the equal energy rule can be applied and the following equation is obtained.

$$R_\mu = \frac{V_e}{V_y} = \sqrt{2\mu - 1}, \quad d_p = \frac{\mu}{\sqrt{2\mu - 1}}d_e$$ (7.2.89)

For long-period systems ($T > 0.5s$), the equal displacement rule can be applied and the following equation is obtained.

$$R_\mu = \frac{V_e}{V_y} = \mu, \quad d_p = d_e$$ (7.2.90)

where $V_e$ is the maximum elastic force response, $V_y$ is the yield strength of an elastoplastic system, $d_e$ is the maximum elastic displacement response and the ductility factor $\mu$ is the ratio of maximum acceptable displacement of the inelastic system to the yield displacement.

It should be noted that the two design criteria are not always conservative and may vary due to input earthquake motions.

Nelson, et al.\(^{55}\) carried out seismic response analyses using a large number of recorded strong earthquake motions around the world for elastoplastic single-degree-of-freedom systems. Statistical processing of the results was carried out. The results are shown in Fig.7.2.29. The figure is partly amended to be used in design. $T$ is the initial period of single-degree-of-freedom system, and $T_g$ is the corner period in the acceleration response spectrum of input earthquake motion (see Fig.7.2.30). The solid line and dashed line in the figure show the 50% (median) and 16% (median$+1\sigma$) exceedance probability of acceptable ductility factor ($\mu = 3$). The lines corresponding to the equal energy rule and the equal displacement rule are also shown.

The figure indicates that the solid line in the long period region (longer than $T/T_g = 1.5$) is in good agreement with the predictions of equal displacement rule. A step increase in $R_\mu$ is observed within the range of $T/T_g = 0.6 \sim 0.7$. The value of $R_\mu$ in the short period range is considerably lower than the predictions of the equal displacement rule. In these short period range, a building with very high strength can have inelastic response displacement less than the acceptable value.

Moreover, when the equivalent static force is evaluated with this recommendation, $R_\mu$ can be calculated using $T/T_g$ shown in Fig.7.2.29.
2) Structural characteristic factor $D_s$ of the current building code regulation

The structural characteristic factor $D_s$ used in the current regulation of the Building Standard Law is calculated according to the ductility of the building. $D_s$ is calculated by using ductility factors and shear force ratios of structural elements constituting a building such as columns, beams, shear-walls and braces. $D_s$ is assigned a value of 0.25~0.5 for steel and steel-reinforced-concrete buildings, and 0.3~0.55 for reinforced concrete buildings. Detailed description of $D_s$ can be obtained from Reference\(^{(17)}\).

(2) Estimation of maximum response displacement

It is necessary to calculate the inelastic response displacement in order to verify that the global safety of building and member damage are within the acceptable limit.

The method of estimating the maximum inelastic response displacement using equivalent single-degree-of-freedom system is described below. If static pushover analysis based on inelastic characteristics of structural members is carried out for calculating the nonlinear force-displacement relationship of equivalent-single-degree-of-freedom system, the results of that can be utilized for the verification.

1) The method of FEMA356

The nonlinear static procedure (NSP) contained in FEMA356\(^{(56)}\) is described below. This method estimates the inelastic maximum displacement using elastic displacement and other modification factors. FEMA356 is a guideline for seismic retrofit, and NSP is the procedure of seismic performance evaluation of buildings that uses nonlinear static analysis.

The relationship between base shear and lateral displacement of equivalent single-degree-of-freedom system is calculated using analytical model that represents the nonlinear load-deformation relationship of each structural element. The lateral displacement of the equivalent single-degree-of-freedom system represents that of the roof level, furthermore the base shear-displacement curve is idealized as bilinear. The maximum inelastic response displacement (target displacement) is calculated using the following equation.

$$\delta_t = C_0 \ C_1 \ C_2 \ C_3 \ S_a \ \frac{T_e^2}{4\pi^2 \ g}$$  (7.2.91)
7.2. Calculation of Seismic Loads

In other words, the maximum inelastic response displacement is predicted by multiplying the maximum displacement \( S_{ag}T_e^2/(4\pi^2) \) of an elastic system with effective fundamental natural period \( T_e \) by the modification factors \( C_0, C_1, C_2, \) and \( C_3 \).

In Eq.(7.2.91), \( C_0 \) is the modification factor which represents the relationship between single-degree-of-freedom system and multi-degree-of-freedom system.

\( C_1 \) is the modification factor which represents the relationship between the elastic and inelastic maximum displacements and is calculated by the following equation.

\[
C_1 = \begin{cases} 
1.0 & T_e \geq T_s \\
1.0 + (R - 1)T_s/T_e & T_e < T_s 
\end{cases} 
\]

(7.2.92)

The effective fundamental natural period \( T_e \) of the building is calculated using the following equation.

\[
T_e = T_i \sqrt{\frac{K_i}{K_e}}
\]

(7.2.93)

where \( T_i \) is the elastic period, \( K_i \) and \( K_e \) are the initial stiffness and effective stiffness of the idealized in bilinear model. \( T_s \) is the corner period (Fig.7.2.30) of the acceleration response spectrum of input earthquake motion. \( R \) is the ratio of the maximum elastic force response to the yield strength of bilinear system and corresponds to the strength reduction factor. \( R \) is calculated using the following equation. The relationship between \( C_1 \) and \( T_e \) is shown in Fig.7.2.31.

\[
R = \frac{S_aW}{V_y}C_m
\]

(7.2.94)

where \( V_y \) is the yield strength of the bilinear model, \( W \) is the weight of the building, \( C_m \) is the modification factor to consider the effect of the higher mode, and \( S_a \) is the spectral response acceleration at \( T_e \). The response acceleration spectrum for design is provided by the U.S. Geological Survey. \( C_2 \) is a modification factor which represents the effect of the hysteretic model on the maximum response. \( C_3 \) is a modification factor which represents the influence of the dynamic P-Δ effect, and is calculated using the following equation.

\[
C_3 = 1.0 + \frac{|\alpha| (R - 1)^{3/2}}{T_e}
\]

(7.2.95)

where \( \alpha \) is the ratio of post-yield stiffness to effective elastic stiffness.

The values of \( C_0, C_2, \) and \( C_m \) which are determined according to the performance level, number of stories, seismic-force-resisting system, etc. are also indicated in the reference and are omitted here.
2) The method of the revised Enforcement Order of the Building Standard Law

The response spectrum method based on equivalent linear system is described in the revised Enforcement Order of the Building Standard Law in 2000, and the maximum response of the building can be calculated using this method. Although the strength reduction factor is not directly used in this method, it is well known for calculating the maximum response displacement.

In this method, the static nonlinear pushover analysis of a building is carried out, and the force-displacement curve for the equivalent single-degree-of-freedom system is evaluated. The maximum response displacement is evaluated as the intersection of this curve and the design spectrum. The detailed description of this method is explained in Reference 17).

7.2.4 Amplification factor due to structural irregularities of the building

When a building having structural irregularities in plan and elevation is subjected to a strong ground motion, the damage may concentrate on a specific portion of the building. This is also pointed out in the reports of the past earthquake damage. The concentrated damage leads to lesser seismic resistance than the uniformly distributed damage.

The design seismic shear for these building is increased by an amplification factor $k_F$ related to the irregularities of the building. Furthermore, for buildings where large concentrated damage is expected, seismic performance will be evaluated by time history response analysis using a suitable analytical model as described in Sec.7.1.1. In the following subsection, the relationship between the structural irregularities in plan or elevation and seismic performance of the building is described.

(1) $F_{es}$ in Ministry of Construction notification

In the current building code$^{[17]}$, $F_{es}$ is used as an amplification factor for the required story capacity based on the irregularities in plan and elevation of the building. $F_{es}$ is obtained by multiplying $F_s$ by $F_e$. $F_s$ is calculated according to the ratio of story stiffness distribution in elevation ($R_s$), $F_e$ is calculated according to the ratio of eccentricity in plan ($R_e$). $R_s$ and $R_e$ are calculated based on the elastic stiffness of structural elements of the building.

Since the damage due to strong earthquake motion is caused by inelastic behavior, an index related to strength is more suitable in representing damage concentration than the index based on elastic
7.2. Calculation of Seismic Loads

There are other guidelines and research using the index related to strength, however, these are not proposed in the form which can be directly used as $k_{fi}$. $F_{es}$ which has generally common values may be used as $k_{fi}$.

**(2) Influence of the collapse mechanism of the building**

The relationship between collapse mechanism and structural characteristic factor is described below. Asari, et al.\(^\text{57}\) performed seismic response analysis using a finite rotation model and several recorded strong earthquake motions. The analytical models considered three kinds of collapse mechanism which are called beam collapse type (SCWB: strong column weak beam model), first story collapse type (SFS: soft first story model) and arbitrary story collapse type (MS: multi story model). The extreme value of the structural characteristic factor was obtained by calculating the base-shear coefficient $C_y$ in the case of collapse due to dynamic $P$-$\Delta$ effect. In the study, the elastoplastic model to represent the hysteretic behavior of elements was applied, and the natural period of the analytical model was varied.

The analytical results are shown in Fig.7.2.32. The $C_y/C_e$ spectra ($C_e$ is the base-shear coefficient of elastic system) shows the mean of the extreme values of the structural characteristic factor based on many earthquake motions. The value of $D_s=0.25$ and $1/R=1/8$ in the figure are the minimum values of the structural characteristic factor of the building code of Japan and the U.S. The extreme value of the structural characteristic factor in the case of first story collapse type (SFS model) and arbitrary story collapse type (MS model) increases almost linearly as the period increases, and it becomes difficult to reduce the design seismic shear with structural characteristic factor for these collapse types. On the other hand, the extreme value of structural characteristic factor for beam collapse type (SCWB model) is about $1/8$ even if the period becomes 4.0s. The structural characteristic factor greatly depends on the collapse mechanism of the building.

The ratio of the extreme value of the structural characteristic factor for specific story collapse type to that of beam collapse type, i.e., the shape factor considering inelastic behavior are shown in Fig.7.2.33. $pD_s$, $mD_s$ and $bD_s$ represent $C_y/C_e$ for the first story collapse type (SFS model), arbitrary story collapse type (MS model), and beam collapse type (SCWB model), respectively. Since the values of the shape factor for collapse mechanism with damage concentration becomes large, it is suggested that the amplification factor related to these irregularities be increased.

In the structural design of the whole collapse mechanism type where the damage will not concentrate to one story, e.g. the beam collapse mechanism, the building is designed to ensure the intended collapse mechanism by preventing brittle shear failure of structural members and by giving appropriate capacity ratios between beams and columns. For the building where the story collapse is expected, the story strength and stiffness must be designed with appropriate vertical distribution. In fact, many buildings have unfavorable distribution due to the restrictions on architectural planning, etc. A typical example is the weak soft first story of low to medium rise buildings which suffered serious damage during the 1995 Hyogoken-Nanbu Earthquake. Although the maximum value of $F_s$ was increased from 1.5 to 2.0 after the 1995 Hyogoken-Nanbu Earthquake, the possibility of inadequacy is pointed out even if the maximum value of 2.0 is applied for these buildings.\(^\text{57}\) Some recommendations such as extending upper shear walls to the weak soft first story are described in Reference.\(^\text{17}\).
(3) Influence of torsional vibration

Torsional vibration using the floor eccentricity of the building is described below. The case of the torsionally unbalanced (in a single direction: mono-asymmetric) multistory building is described. The ratio of eccentricity $R_e$ used in the current code is defined by the so-called story eccentricity\(^{17}\).

1) Static torsional moment

The distribution of static torsional moment is described below by referring to Reference\(^{58}\). The building model has $N$ stories and $L$ frames. The story eccentricity $e_i$ defined as the distance between the center of mass and the center of stiffness at the $i$-th story is obtained as follows.

$$e_i = (x_m)_i - (x_R)_i \quad (7.2.96)$$

where $(x_m)_i$ is the centroid of loads at the story, and $(x_R)_i$ is the center of each frame reaction $f_{ij}$ calculated as follows.

$$ (x_R)_i = \frac{\sum_{j=1}^{L} f_{ij} x_j}{\sum_{j=1}^{L} f_{ij}} \quad (7.2.97)$$

where $x_j$ is the x-coordinate of each frame location. The floor torque (torsional moment) $T_i$ of the $i$-th story is calculated as follows, where $P_i$ is the resultant of the reaction forces.

$$T_i = P_i e_i = e_i \sum_{j=1}^{L} f_{ij} \quad (7.2.98)$$

Furthermore, the torsional moment $(M_t)_k$ of the $k$-th story is obtained as follows.

$$ (M_t)_k = \sum_{i=k}^{N} T_i \quad (7.2.99)$$

From the above relationships, it is found that the torsional moment at each story is influenced by the vertical distribution of lateral forces $P_i$ at each floor.
2) Equation of motion of multistory building with mono-asymmetry

The equation of translational and torsional motion using shear-type model of multistory building with mono-asymmetry when earthquake motion acts from one horizontal direction is described as follows.

\[
m_i (\ddot{u}_i + \ddot{u}_g) + F_i = 0 \tag{7.2.100}
\]

\[
I \ddot{\theta}_i + F_{\theta i} + F_i e_i = 0 \tag{7.2.101}
\]

where \(F_i\) is the resultant of frame reactions at the \(i\)-th story, \(F_{\theta i}\) is the resisting moment around the center of stiffness of frame reactions, \(I\) is the rotational moment of inertia about the center of mass, \(u_i\) and \(\theta_i\) represent the response values of translational and torsional components, respectively.

From Eq.(7.2.101), \(F_i e_i\) is the torsional moment (torque) due to translational response at the \(i\)-th story which causes the torsional response. When \(F_i\) or \(e_i\) is large, i.e., input earthquake motion or the eccentricity is large, the torsional response will become large, a difference in the torsional displacement between time history and static analyses will be obtained. On the other hand, it is also found that decreasing eccentricity and increasing torsional stiffness are effective in decreasing the torsional response. Furthermore, in many cases, the influence of eccentricity may become large as the damage at the flexible side elements of a story increases due to inelastic torsional response. In the current recommendations, when the influence of torsional vibration is expected to be large, three-dimensional time history response analysis using suitable model will be carried out to estimate torsional response as described in Sec.7.1.1. When there is structural irregularities in plan and elevation of the building, damage concentration will be generated even if design seismic shear is increased by this amplification factor. Furthermore, when the uncertainty of earthquake motion or the probability of occurrence of earthquake motion having higher intensity than design level is considered, it is important to ensure the ductility of the story where large damage concentration is expected.

7.3 Design Earthquake Motions

7.3.1 Fundamental concept for generating design earthquake motions

It is recognized that the main purposes of building design are both to save human lives and to protect the property. For the latter purpose, the maximum probable earthquake motion of at least once in the design service life of the building is defined, and most of the main structures should stay within the elastic limit and no structural damage is allowed. For the former purpose, the maximum credible earthquake motion from the engineering viewpoint is defined at the site, and no harmful damage to the building is allowed. In practice, in order to design important buildings, design earthquake
motions are firstly determined and dynamic earthquake response analyses are performed to ensure safety performance of buildings against the seismic hazard.

The characteristics of earthquake ground motions depend on the seismic source, the propagation path of seismic waves and the local site conditions. Fig.7.3.1 shows a schematic diagram of these factors. Recently, a significant amount of valuable knowledge in seismology, earthquake engineering, geology, etc., have been achieved, which provide useful information for evaluating earthquake motions. A design earthquake motion should reflect natural phenomena as much as possible that may affect a building in the future. Therefore, it should be evaluated based on the up-to-date knowledge and consistent philosophy, not far from the scientific basis but within acceptable engineering limits. From the above-mentioned point of view, this chapter proposes a rational methodology of evaluating site-specific design ground motions based on site-specific scenario earthquakes.

![Fig.7.3.1 Effects of source, path and site conditions on ground motions](image)

Fig.7.3.2 shows the methodology of evaluating design earthquake motion. The procedures to generate the design earthquake motions are classified into two types as follows; a) generation of a design earthquake motion that is compatible with the target spectrum (see Sec.7.3.2), and b) generation of a design earthquake motion based on a scenario earthquake considering local site conditions and properties of the building (see Sec.7.3.3).
7.3. Design Earthquake Motions

![Flowchart for Sec.7.3.2](image)

![Flowchart for Sec.7.3.3](image)

**Fig.7.3.2** Methodology of evaluating design earthquake motion

7.3.2 Design earthquake motions compatible with the design response spectrum

First, a time history is generated applying Fourier Transform to the initial Fourier amplitudes and Fourier phases that are assumed in advance. In case the initial Fourier phases are assumed to be random, the design envelope function is applied to the time history. The Fourier amplitudes are modified iteratively in order to make the response spectrum of the time history compatible with the design response spectrum (the target spectrum), and the design earthquake motion is generated\(^59\).

7.3.3 Design earthquake motions based on the scenario earthquakes

As shown in the right side of Fig.7.3.2, roughly speaking, the procedure to generate the design earthquake motion is composed of the following three steps; evaluation of the scenario earthquakes, evaluation of earthquake motions at the construction site which are induced by the scenario earthquakes, and evaluation the design earthquake motions\(^60,61\). The design lifetime is determined considering the demand of the building owner, the building use, daily function and the law related to depreciation. The seismic source, the propagation path of seismic waves and the local site conditions should be examined as much as possible to evaluate the design earthquake motion. Here the design motion is defined at the free field on the bearing stratum that supports the building.

Fig.7.3.3 shows a schematic explanation of classification of the earthquakes. First, the earthquakes considered herein are identified based on the knowledge in seismology, earthquake engineering geology, etc. The earthquakes under consideration are classified into the following four types; an interplate earthquake, an oceanic intra-plate or intra-slab earthquake, a continental intra-plate (crustal) earthquake, and a right-under-site intra-plate (crustal) earthquake. The location, magnitude and recurrence time of each earthquake are determined by considering plate tectonics, historical earthquakes, active faults, seismotectonics, microearthquakes, layered structure, etc.\(^62,63,64,65,66,67,68,69,70,71,72,73,74\)
Next, the characteristics of the seismic source, the propagation path of seismic waves, and the local site conditions are taken into account, then the earthquake motions induced by the scenario earthquakes are evaluated at the site. Fig.7.3.4 and Table 7.3.1 show the flowchart and explanation of methods to evaluate the time histories of earthquake ground motions. Here, several evaluation methods are prepared: use of a strong motion record of the scenario earthquake at or near the site, a theoretical approach based on fault models and elastic waves, an empirical or statistical Green’s function approach based on fault models, a hybrid approach based on a theoretical approach in longer period range and on a semi-empirical approach with Green’s function in shorter period range, and an empirical approach by using the standard spectrum and the standard duration of earthquake motions\(^75, 76\). This methodology adopts, in an integrated manner, the most adequate evaluation method according to the quality and quantity of the available information, because the feasibility and the reliability of such evaluation depend highly on the available information. By using the adopted method, time histories of the earthquake motions are evaluated.

Last, the design earthquake motions are determined by judging the evaluated earthquake motions. The most adequate way is selected from the following three; using all the evaluated motions, using the representative evaluated motions, or generating new design motions by using the target spectrum and the duration based on the characteristics of the evaluated motions.

**Table 7.3.1** Modelling of source, path and site effects in each method to evaluate earthquake ground motions

<table>
<thead>
<tr>
<th>Method to Evaluate Earthquake Ground Motions</th>
<th>Source (Earthquake)</th>
<th>Path (Propagation)</th>
<th>Site (Local)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong Motion Record of Scenario Earthquake</td>
<td>Record</td>
<td>Record</td>
<td>Record</td>
</tr>
<tr>
<td>Theoretical Method</td>
<td>Theory</td>
<td>Theory</td>
<td>Theory or statistics</td>
</tr>
<tr>
<td>Hybrid method</td>
<td>Theory &amp; Statistics</td>
<td>Theory &amp; Record or Statistics</td>
<td>Theory &amp; Record or Statistics</td>
</tr>
<tr>
<td>Empirical Green’s Function Method</td>
<td>Theory &amp; Record</td>
<td>Record</td>
<td>Record</td>
</tr>
<tr>
<td>Statistical Green’s Function Method</td>
<td>Theory &amp; Statistics</td>
<td>Statistics</td>
<td>Theory or Statistics</td>
</tr>
<tr>
<td>Empirical method</td>
<td>Statistics</td>
<td>Statistics</td>
<td>Statistics</td>
</tr>
</tbody>
</table>
7.3. Design Earthquake Motions

Assuming a Scenario Earthquake

Strong Motion of the Scenario Earthquake is Recorded at or near the Construction Site

Using the Fault Model of the Scenario Earthquake

Underground Information is Sufficient in order to Model the Propagation and Site Effects

Modeling the Irregular Media

Strong Motion Record of Scenario Earthquake

Theoretical Method for Irregular Media

Theoretical Method for Horizontally Layered Medium

Hybrid Green's Function Method

Hybrid Method

Empirical Green's Function Method

Statistical Green's Function Method

Empirical Time History Generation Method

Time History of Earthquake Ground Motion

Quality and Quantity of Information Used for Strong Motion Evaluation

| High | | | | Low |

Possibility of Practical Use in Long Period Motion Evaluation

| High | | | | Low |

Possibility of Practical Use in Broadband Motion Evaluation

| High | Low | Low | High | Low |

Possibility of Practical Use in Short Period Motion Evaluation

| High | Low | | | | High |

Possibility of Practical Use in Spatial Motion Evaluation

| Low | Middle | Middle | Low | Middle | High |

Fig. 7.3.4 Flowchart and explanation of motions to evaluate earthquake ground motions
References (Chapter 7)

1) International Organization for Standardization : ISO 2394 General principles on reliability for structures, 1998.6
References


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